A MATHEMATICAL ANALYSIS OF A THREE PHASE INDUCTION MOTOR, OPERATED THREE PHASE AND SINGLE PHASE WITH UNBALANCED ROTOR CONDITIONS

12

A THESIS

Presented to

the Faculty of the Division of Graduate Studies

Georgia School of Technology

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Electrical Engineering

by

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December, 1947

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Approved:

Date Approved by Chairman Dec. 17, 1947

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I THE PROBLEM

The purpose of this thesis is to compare the results obtained by mathematical analysis with actual laboratory tests on a three phase induction motor operated three phase and single phase with unbalanced rotor conditions.

Much theory on induction motors has been developed, but the method of analysis employing symetrical components best suits the particular problem at hand, and this theory will be used throughout the thesis.

DEFINITION OF SYMBOLS

I_s --------------- Stator Current

I_R --------------- Rotor Current

E_s --------------- Stator Line Voltage

E_R --------------- Rotor Line Voltage

E_g --------------- Rotor Induced Voltage

E_{an} , E_{bn} , E_{cn} ---- Rotor Phase Voltage

R_s --------------- Stator Resistance

R_R --------------- Rotor Resistance Referred to Stator

X_s --------------- Stator Reactance

X_R --------------- Rotor Reactance Referred to Stator

Z_a , Z_b , Z_c , Z_R - External Rotor Circuit Impedances

$I\phi$ --------------- No Load Current

s --------------- Machine Slip

a --------------- Operator $= e^{-j120°}$

A Subscript "1" Designates Positive Sequence Component.

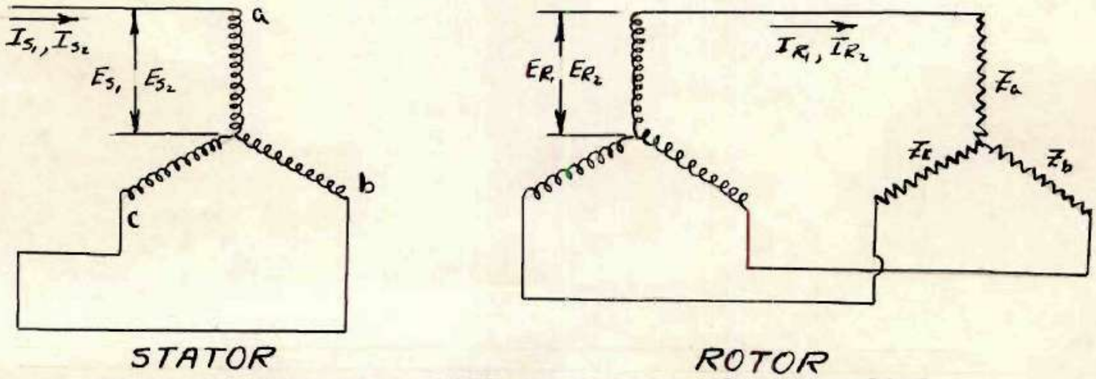
A Subscript "2" Designates Negative Sequence Component.

A Subscript "0" Designates Zero Sequence Component.

II DEVELOPEMENT OF THEORY FOR THREE PHASE OPERATION

A balanced three phase voltage applied to a circuit may be considered as consisting of a positive and negative component. Of course the negative sequence voltage will be zero, but it is carried through symbolically in the developement of the theory.

Generally, the motor may be pictured as below:



First consider the positive sequence voltage applied to the stator. Currents fo fundamental frequency in the stator will give rise to currents of slip frequency in the rotor. An unbalance in the rotor will cause these currents to be unbalanced. These unbalanced currents of slip frequency in the rotor may be resolved into positive and negative sequence currents. The later sets up fields which rotate negatively with respect to the rotor at slip frequency.

Those currents in turn induce currents in the stator of frequency equal to the fundamental frequency minus twice slip frequency.

With E_{s_1} applied to the stator the resultant air gap flux, rotating at synchronous velocity induces E_{g_1} in the stator and sE_{g_1} in the rotor, where s is taken as a fraction. Therefore, in the stator

$$E_{s_1} = E_{g_1} + (R_s + jX_s) I_{s_1} \quad ,$$

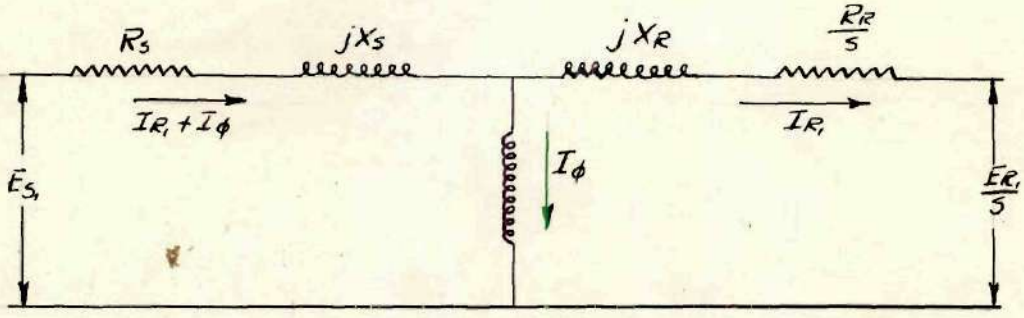
and in the rotor

$$sE_{g_1} = E_{R_1} + (R_R + jX_R) I_{R_1}$$

Dividing through by s,

$$E_{g_1} = \frac{E_{R_1}}{s} + jX_R I_{R_1} + \frac{R_R}{s} I_{R_1}$$

Associating the magnetizing current of the machine with the positive sequence current, the equivalent circuit below may be drawn.



This diagram agrees with the equations derived. From it a more suitable relation may be set up as follows:

$$\frac{E_{R_1}}{s} = E_{S_1} - (R_S + jX_S)(I_{R_1} + I_\phi) - (R_R + jX_R)I_{R_1}$$

$$\text{or} \quad E_{R_1} = sE_{S_1} - [sR_S + R_R + js(X_S + X_R)]I_{R_1} - s(R_S + jX_S)I_\phi$$

In the developement of the negative sequence equivalent circuit, assume a negative sequence, voltage, E_{R_2}, of slip frequency applied to the rotor and assume the rotor to be stationary and the stator rotating with a speed (1-s) in the negative direction. This will give the same result as the stator stationary and the rotor rotating with a speed (1-s). The applied voltage sets up a flux which rotates with a velocity (-s), and since the assumed velocity of the stator is -(1-s), these velocities will be equal for s 0.5. For this slip no voltages are induced in the stator and the characteristics correspond to that of an ordinary induction motor operating at synchronous speed. For s greater than 0.5 the negative velocity of the stator is greater than the negative velocity of the flux set up by the negative sequence voltage and the machine acts like an induction generator. This is the condition present when a negative sequence voltage is applied to the stator of an induction motor; that is, it produces negative torque as compared with the torque produced when a positive sequence voltage is applied.

With this voltage, E_{R_2}, applied to the rotor and current flowing out of the rotor taken as positive, the following relationship may be written,

$$E_{R_2} = E_{g_2} - (R_R + jX_s)I_{R_2}$$

The speed of the flux which produces E_{g_2} is (-s) relative to the rotor; (1-2s) relative to the stator. This flux therefore induces the voltage $-(1-2s)E_{g}$, in the stator windings. Since the negative sequence voltage in the stator is zero, this voltage must be equal to the impedance drop of the stator at the frequency (1-2s)f. Also, with all machine constants referred to the stator, $I_{S_2} = I_{R_2}$ and

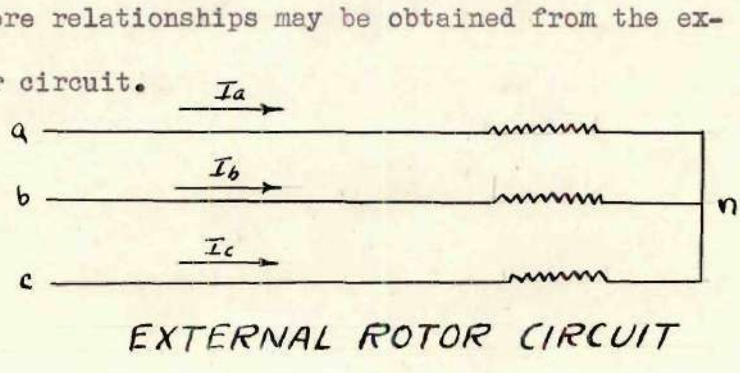
$$-\left(\frac{1-2s}{s}\right)E_{g_2} = R_s(-I_s) - j(1-2s)x_s(-I_{R_2})$$

$$\text{or} \qquad E_{g_2} = \left[\frac{s}{1-2s}R_s - jsx_s\right]I_{R_2}$$

Substituting this value in the equation for E_{R_2};

$$E_{R_2} = -\left[R_R - \frac{sR_s}{1-2s} + js(x_s + x_R)\right]I_{R_2}$$

We now have two equations involving E_{R_1}, E_{R_2}, I_{R_1} and I_{R_2}. Two more relationships may be obtained from the external rotor circuit.



EXTERNAL ROTOR CIRCUIT

From this diagram the following relationships are true:

$$E_{an} = Z_a I_a$$

$$E_{bn} = Z_b I_b$$

$$E_{cn} = Z_c I_c$$

Since any current is equal to the sum of its symetrical components,

$$E_{an} = Z_a (I_{R_1} + I_{R_2})$$

$$E_{bn} = Z_b (a^2 I_{R_1} + a I_{a_2})$$

$$E_{cn} = Z_c (a I_{R_1} + a^2 I_{R_2})$$

Also,

$$E_{an} = E_{an_1} + E_{an_2}$$

$$E_{bn} = a^2 E_{an_1} + a E_{an_2}$$

$$E_{cn} = a E_{an_1} + a^2 E_{an_2}$$

Multiplying the second of these equations by a, and the third by a^2, and adding the three equations together,

$$3 E_{an_1} = E_{an} + a E_{bn} + a^2 E_{cn}$$

or

$$E_{an_1} = 1/3 (E_{an} + a E_{bn} + a^2 E_{cn})$$

Likewise,

$$E_{an_2} = 1/3 (E_{an} + a^2 E_{bn} + a E_{cn})$$

By substitution from above,

$$E_{an_1} = 1/3 \left[Z_a (I_{R_1} + I_{R_2}) + a Z_b (a^2 I_{R_1} + a I_{R_2}) + a^2 Z_c (a I_{R_1} + a^2 I_{R_2}) \right]$$

Collecting terms,

$$E_{a_1} = \tfrac{1}{3}(Z_a + Z_b + Z_c)I_{R_1} + \tfrac{1}{3}(Z_a + a^2 Z_b + a Z_c)I_{R_2}$$

But,

$$\tfrac{1}{3}(Z_a + Z_b + Z_c) = Z_{a_0}$$

$$\tfrac{1}{3}(Z_a + a^2 Z_b + a Z_c) = Z_{a_2}$$

Therefore,

$$E_{a_1} = Z_{a_0} I_{R_1} + Z_{a_2} I_{R_2}$$

Likewise,

$$E_{a_2} = Z_{a_1} I_{R_1} + Z_{a_0} I_{R_2}$$

We now have four equations and four unknowns which, after substituting numerical values for known quantities, may be solved. Since these are vector equations the line current and power factor may be obtained from them.

III DEVELOPEMENT OF THEORY FOR SINGLE PHASE OPERATION

Generally the same theory will be applied to the machine when operated single phase as when operated three phase.

The rotor circuit remains the same. Therefore we may use the second two equations developed in the three phase theory:

$$E_{R_1} = Z_{a_0} I_{R_1} + Z_{a_2} I_{R_2}$$

$$E_{R_2} = Z_{a_1} I_{a_1} + Z_{a_0} I_{R_2}$$

To adapt the conditions under single phase operation to the three phase theory, the applied single phase voltage must be broken up into symetrical components. There will be no zero sequence component of current present when the machine is operated single phase, since there is no path for it to flow. It follows then that there is no zero sequence component of voltage applied. In order for this to be true, the applied voltage must be considered as being made up of two voltages equal and 180° out of phase. Call these voltages E_A and E_C. Then the symetrical components will be

$$E_{A_1} = 1/3\left[E_A + a^2 E_C\right]$$

$$E_{A_1} = 1/3\left[E_A + E_C \angle 60^\circ\right]$$

and

$$E_{A_2} = 1/3\left[E_A + E_C \overline{|60^\circ}\right]$$

Referring to the three phase theory the following equations may be written:

$$E_{R_1} = SE_{S_1} - [SR_S + R_R + jS(X_S + X_R)]I_{R_1} - S(R_S + jX_S)I\phi_1$$
$$E_{R_2} = SE_{S_2} - [(R_R - \frac{SR_S}{1-2S}) + jS(X_S + X_R)]I_{R_2} + S[\frac{R_S}{1-2S} - jX_S]I\phi_2$$

In the above equations $I\phi_1$ is equal to the no load current when the motor is operating three phase. $I\phi_2$ is equal to the no load current when the motor is operating single phase minus $I\phi_1$.

These four equations may be solved and the line current and power factor obtained.

IV TECHNIQUE OF LABORATORY EXPERIMENTS

The machine tested was a five horsepower, three phase, wound rotor, induction motor equipped with slip rings to the rotor. In the three phase runs the motor was supplied by an alternator driven by a synchronous motor. The rating of the alternator was the same as that of the motor so therefore the motor could not be overloaded safely to any great degree. When run single phase, the motor was supplied by the incoming power line.

The following tests were made on the motor:

1. Ratio of Transformation Test.

The stator windings were connected to their rated voltage, V_1, and the rotor voltage E_2 was found. This voltage E_2, was then impressed on the rotor and the stator induced voltage, V' was read. The ratio of transformation is then

$$a = \frac{V}{E_2}\sqrt{\frac{V}{V'}}$$

2. Stator and Rotor Resistance.

These constants were measured with a resistance bridge.

3. <u>No Load Test.</u>

The motor was run at no load on balanced three phase voltage of varying magnitude. From this test the windage and friction and core loss was calculated.

4. <u>Blocked Rotor Test.</u>

The rotor of the machine was held stationary while reduced voltages were applied to the stator. From this test the equivalent resistance and reactance of the motor was found.

5. <u>Three Phase Load Test.</u>

Several combinations of unbalanced resistances were connected to the slip rings. For each combination a load run was made on the motor by means of a prony brake. The applied voltage was held constant and stator current, watts input, torque and speed were read for each load point.

6. <u>Single Phase Load Test.</u>

The rotor was unbalanced as in the three phase tests. The motor was started with a three phase voltage and when it reached rated

Speed one of the three leads to stator was disconnected. Loading was done with a prony brake and readings made as in the three phase tests.

V SUMMARY

At the outset of the experimentation it was expected, that with an unbalanced rotor, the line currents to the machine would be unbalanced. In the load runs this condition did not appear even with the most extreme unbalance in the rotor circuit. The motor was broken down several times in an effort to uneqalize the stator currents.

The unbalance, although desired, was not absolutely necessary to illustrate the mathematical analysis of the machine performance. The analysis shows that the stator currents contain so small a negative component that it can not be read on the meters used. In the rotor the ratio of positive to negative sequence current usually ran about fifteen to one. Due to the high no load current this ratio in the stator is about twenty to one. Possibly a larger machine with lower no load current would draw line currents which were sufficiently unbalanced to be read on the amp-meters.

For both the three phase and single phase runs the calculated points matched the test points closely enough to prove the analysis a valid one. The calculated horsepower output was always a little high. The curves of the efficiency, power factor and line current plotted against this horsepower, although extended, match the test curves very nicely.

BIBLIOGRAPHY

Dahl, O. G., Electrical Circuits, Theory and Applications New York: McGraw-Hill Book Company, Inc., 1938.

Lawrence, R. R., Principles of Alternating Current Machinery. New York: McGraw-Hill Book Company, Inc., 1939.

Lyon, W. V., Applications of the Methods of Symetrical Components, New York: McGraw-Hill Book Company, Inc., 1937.

Wagner, C. F. and R. D. Evans, Symetrical Components, New York: McGraw-Hill Book Company, Inc., 1933.

Puchstein, A. F. and T. C. Lloyd, Alternating Current Machines, New York: John Wiley and Sons, Inc., 1942

NO LOAD TEST

V	I_1	I_2	I_3	W_{12}	W_{23}	W
220	8.8	8.8	9.0	720	1220	500
200	7.92	7.8	8.0	560	1008	440
180	7.0	7.0	7.0	440	816	376
160	6.2	6.2	6.2	328	648	320
140	5.4	5.4	5.4	224	512	288
118	4.6	4.6	4.6	136	392	256
100	4.0	4.0	4.0	80	288	208

BLOCKED ROTOR TEST

V	I_1	I_2	I_3	W_{12}	W_{23}	W	CAL. $I_{sc} \cos\theta$
52	19.6	19.6	19.6	908	60	968	10.5
43	16.0	16.0	16.0	632	40	672	9.0
33	12.0	12.0	12.0	372	20	392	6.8

BALANCED 3 ϕ - 220 V. STATOR

SHORT CIRCUITED ROTOR

I_s	W_1	W_2	W_T	RPM	TORQUE	H.P.	SLIP(%)	EFF(%)	P.F(%)
8.9	1170	-720	450	1190	0.0	0.00	8.83	----	13.3
10.0	1830	-150	1680	1180	6.2	1.37	1.66	60.5	44.0
12.5	2570	550	3120	1150	14.2	3.10	4.17	74.0	65.5
14.8	3220	1090	4310	1130	20.7	4.45	5.83	77.0	67.5
17.5	3820	1550	5370	1120	26.1	5.54	6.67	77.0	80.5
20.0	4400	1970	6370	1110	30.8	6.50	7.50	76.2	83.5

BALANCED 3 ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.38,0.38,1.25 OHMS

TEST DATA

I_s	W_1	W_2	W_T	RPM	TORQUE	SLIP(%)	P.F(%)	EFF(%)	H.P.
8.9	1170	-720	450	1190	0.0	0.83	13.3	----	----
9.2	1480	-500	980	1160	2.9	3.33	27.9	49.5	0.65
11.0	2190	170	2360	1110	10.6	7.50	56.3	70.8	2.24
12.8	2670	580	3250	1080	15.4	10.00	66.7	72.9	3.17
13.9	2940	810	3750	1060	19.0	11.67	67.5	76.3	3.84
14.7	3120	890	4010	1040	20.0	13.33	71.6	73.4	3.96
15.5	3330	1030	4360	1020	21.4	15.00	73.8	71.3	4.17

CALCULATED POINTS

I_s	P.F.(%)	EFF(%)	H.P.
8.9	10.5	----	0.00
9.0	27.1	55.0	0.75
10.0	56.2	74.5	2.42
12.2	69.0	75.0	3.41
14.9	71.3	73.6	3.79
16.9	75.4	68.6	4.31

BALANCED 3 ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.38,0.38,0.73 OHMS

TEST DATA

I_s	W_1	W_2	W_r	RPM	TORQUE	SLIP(%)	P.F(%)	EFF(%)	H.P.
8.9	1170	-720	450	1190	0.0	0.83	13.3	----	----
9.2	1470	-520	950	1170	2.9	2.50	27.1	50.7	0.65
11.2	2270	230	2500	1120	11.0	6.66	58.6	70.6	2.36
12.4	2580	500	3080	1100	14.1	8.43	65.2	71.4	2.95
13.6	2920	770	3690	1070	18.4	10.82	71.3	75.8	3.75
15.0	3200	1000	4200	1050	20.3	12.50	73.5	72.1	4.06

CALCULATED POINTS

I_s	P.F.(%)	EFF(%)	H.P.
9.1	13.3	----	0.00
9.8	28.2	43.6	0.80
11.7	55.1	69.8	2.25
14.1	66.5	75.1	3.05
16.0	73.0	80.0	3.75
16.3	77.4	80.7	4.14

BALANCED 3 ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.19,0.38,1.25 OHMS

TEST DATA

I_s	W_1	W_2	W_T	RPM	TORQUE	SLIP(%)	P.F(%)	EFF(%)	H.P.
8.9	1170	-720	450	1190	0.0	0.83	32.2	53.1	0.81
9.3	1560	-420	1140	1160	3.6	3.33	32.2	70.1	2.26
11.1	2220	180	2400	1120	10.6	6.66	57.3	70.1	2.26
12.4	2530	480	3010	1100	14.5	8.43	63.7	75.6	3.05
13.6	2830	710	3540	1070	17.0	10.82	68.8	73.4	3.48
16.1	3450	1180	4630	1050	22.6	12.50	75.6	72.8	4.52

CALCULATED POINTS

I_s	P.F.(%)	EFF(%)	H.P.
8.1	9.2	----	0.00
8.7	32.5	55.0	1.00
10.0	55.6	73.7	2.28
12.2	68.4	75.4	3.15
13.0	70.0	77.2	3.64
17.5	74.7	70.6	4.46

BALANCED 3 ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.19,0.38,0.73 OHMS

TEST DATA

I_s	W_1	W_2	W_T	RPM	TORQUE	SLIP(%)	P.F(%)	EFF(%)	H.P.
8.9	1170	-720	450	1190	0.0	0.83	13.3	----	----
9.0	1390	-570	820	1170	2.3	2.50	23.9	47.3	0.52
10.8	2120	110	2230	1130	9.5	5.83	54.3	68.5	2.05
12.5	2580	540	3120	1100	14.4	8.43	65.5	72.1	3.02
14.2	3060	870	3930	1080	19.6	10.00	72.7	76.6	4.04
15.1	3240	1030	4270	1060	21.0	11.67	74.3	74.5	4.24
16.0	3460	1200	4660	1050	22.5	12.50	76.5	72.2	4.50

CALCULATED POINTS

I_s	P.F.(%)	EFF(%)	H.P.
8.7	13.6	----	0.00
9.0	27.6	50.0	0.91
11.6	50.1	73.5	2.28
12.0	66.5	75.2	3.36
15.0	71.1	75.1	4.00
14.5	72.2	76.6	4.32
16.8	75.5	73.9	5.61

BALANCED 3 ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.19,0.38,0.38 OHMS

TEST DATA

I_s	W_1	W_2	W_T	RPM	TORQUE	SLIP(%)	P.F(%)	EFF(%)	H.P.
8.9	1170	-720	450	1190	0.0	0.83	13.3	----	----
9.0	1420	-550	870	1150	2.6	4.17	25.4	50.4	0.58
10.5	2050	100	2150	1130	9.3	5.83	53.8	69.7	2.01
12.5	2640	600	3240	1110	14.8	7.50	68.1	72.1	3.13
13.6	2950	820	3770	1090	18.5	9.17	13.3	76.3	3.85
15.6	3400	1200	4600	1070	22.0	10.82	78.0	74.0	4.50
16.3	3540	1310	4850	1060	23.5	11.67	78.2	73.2	4.75

CALCULATED POINTS

I_s	P.F.(%)	EFF(%)	H.P.
7.9	12.1	----	0.00
7.9	28.9	47.3	0.55
10.0	50.0	66.6	1.92
12.0	73.5	76.2	3.18
14.6	74.8	74.8	3.89
15.0	80.0	76.0	4.45
16.8	81.2	72.2	4.87

BALANCED 3 ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.19,0.19,1.83 OHMS

TEST DATA

I_s	W_1	W_2	W_T	RPM	TORQUE	SLIP(%)	P.F(%)	EFF(%)	H.P.
8.9	1170	-720	450	1190	0.0	0.83	13.3	----	----
9.1	1410	-550	860	1170	2.7	2.50	25.1	52.4	0.60
11.4	2240	200	2440	1120	11.1	6.66	56.7	72.3	2.36
12.9	2590	530	3120	1100	14.9	8.33	64.0	74.6	3.12
13.8	2850	690	3540	1090	17.5	9.17	67.4	76.7	3.64
15.9	3320	1040	4360	1050	21.3	12.50	72.0	73.1	4.27

CALCULATED POINTS

I_s	P.F.(%)	EFF(%)	H.P.
9.2	18.0	----	0.00
9.6	30.5	47.7	0.62
12.9	55.0	70.0	2.50
12.4	63.0	75.2	3.25
15.8	69.8	74.2	3.84
17.7	76.3	71.6	4.75

BALANCED 3 ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.19,0.19,0.38 OHMS

TEST DATA

I_s	W_1	W_2	W_T	RPM	TORQUE	SLIP(%)	P.F(%)	EFF(%)	H.P.
8.9	1170	-720	450	1190	0.0	0.83	13.3	----	0.00
9.3	1440	-450	990	1170	2.9	2.50	28.5	48.1	0.64
11.4	2330	290	2620	1140	11.3	5.00	60.3	70.2	2.46
13.7	2960	830	3790	1110	18.4	7.50	72.5	76.5	3.89
15.6	3400	1270	4670	1080	21.9	10.00	78.6	72.0	4.50
19.5	4680	1610	6290	1070	25.0	10.82	84.6	72.0	6.07

CALCULATED POINTS

I_s	P.F.(%)	EFF(%)	H.P.
8.0	10.0	----	0.00
8.6	28.5	43.8	0.86
13.0	66.9	74.1	3.00
13.0	73.9	77.4	3.95
16.1	81.8	75.9	4.75
19.6	86.0	74.0	6.37

BALANCED 3 ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.19,0.19,1.25 OHMS

TEST DATA

I_s	W_1	W_2	W_T	RPM	TORQUE	SLIP(%)	EFF(%)	P.F(%)	H.P.
8.9	1170	-720	450	1190	0.0	-0.83	----	13.3	0.00
9.1	1360	-540	820	1170	2.3	2.50	46.5	23.8	0.51
10.5	2270	250	2520	1120	11.0	6.66	69.4	63.6	2.34
13.6	2780	700	3480	1090	16.8	9.17	74.8	67.7	3.49
15.9	3310	1080	4390	1070	21.0	10.82	72.9	73.0	4.29
18.2	3760	1450	5210	1030	26.0	14.18	71.4	75.5	4.98

CALCULATED POINTS

I_s	P.F(%)	EFF(%)	H.P.
10.0	7.5	----	0.00
10.3	33.1	55.0	0.95
11.3	57.5	67.5	2.25
13.6	68.4	75.0	3.62
15.1	70.8	75.8	4.30
18.6	73.5	66.7	5.15

BALANCED 3 ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.19,0.19,0.73 OHMS

TEST DATA

I_s	W_1	W_2	W_T	RPM	TORQUE	SLIP(%)	P.F(%)	EFF(%)	H.P.
9.0	1170	-720	450	1190	0.0	0.83	13.3	----	0.00
9.4	1500	-360	1140	1170	3.5	2.50	32.2	51.6	0.79
10.9	2150	110	2260	1130	10.0	5.83	55.0	71.1	2.15
12.5	2570	500	3070	1120	14.5	6.66	64.7	75.4	3.10
14.2	3000	880	3880	1090	18.4	9.17	72.2	73.6	3.82
16.2	3420	1250	4670	1070	22.1	10.82	76.7	72.1	4.52
17.5	3710	1470	5180	1050	25.0	12.50	78.6	72.0	5.00
18.7	3970	1670	5640	1040	28.0	13.32	79.6	71.1	5.47

CALCULATED POINTS

I_s	P.F.(%)	EFF(%)	H.P.
7.6	11.4	----	0.00
8.9	31.6	48.2	0.96
11.6	68.6	68.7	2.39
11.3	72.5	72.3	2.90
13.4	77.1	76.5	3.94
14.9	77.9	75.4	4.62
15.6	80.9	75.7	5.13
19.3	82.2	70.7	5.85

BALANCED SINGLE ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.19,0.38,0.73 OHMS

TEST DATA

I_s	W	TORQUE	RPM	H.P.	SLIP(%)	EFF(%)	P.F(%)
13.8	630	0.0	1190	0.00	0.83	----	20.8
14.3	1210	3.6	1180	0.81	1.67	50.0	38.4
16.3	2050	7.5	1140	1.63	5.00	59.3	57.2
17.6	2400	10.2	1130	2.20	5.83	68.4	62.0
18.6	2700	11.3	1120	2.41	6.67	66.6	66.0
19.9	3040	13.9	1100	2.90	8.33	71.2	69.5

CALCULATED POINTS

I_s	H.P.	EFF.(%)	P.F.(%)
12.8	0.00	----	10.4
14.0	0.80	53.0	37.7
16.6	1.72	67.1	56.1
17.4	2.25	70.0	63.9
19.1	2.49	70.2	65.6
21.5	2.98	68.4	68.4

BALANCED SINGLE ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.19,0.38,0.38 OHMS

TEST DATA

I_s	W	TORQUE	RPM	H.P.	SLIP(%)	EFF(%)	P.F(%)
13.8	630	0.0	1190	0.00	0.83	----	20.8
14.2	1100	3.3	1180	0.74	1.66	50.1	35.2
15.6	1760	6.6	1170	1.47	2.50	62.3	51.2
17.0	2260	9.0	1150	1.97	4.16	65.0	60.4
17.6	2420	10.2	1140	2.21	5.00	68.1	62.5
18.4	2640	11.0	1130	2.37	5.83	67.0	65.2
19.4	3000	13.5	1110	2.85	7.50	70.8	70.3

CALCULATED POINTS

I_s	H.P.	EFF(%)	P.F.(%)
11.5	0.00	----	16.2
12.9	0.80	48.1	37.7
14.9	1.40	65.3	47.6
15.9	2.02	71.2	61.5
18.4	2.30	72.1	62.3
18.8	2.45	72.2	65.5
21.0	2.88	69.5	71.5

BALANCED SINGLE ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.19,0.19,1.25 OHMS

TEST DATA

I_s	W	TORQUE	RPM	H.P.	SLIP(%)	EFF(%)	P.F(%)
13.8	630	0.0	1190	0.00	0.83	----	20.8
14.9	1470	4.2	1160	0.93	3.33	47.2	44.8
16.6	2050	8.2	1130	1.61	5.83	58.6	56.2
17.7	2390	10.2	1120	2.17	6.67	67.8	61.4
18.7	2580	11.1	1110	2.35	7.50	68.0	62.8
20.5	3050	13.8	1090	2.86	9.17	70.0	67.7

CALCULATED POINTS

I_s	H.P.	EFF(%)	P.F.(%)
12.2	0.00	----	19.6
13.7	1.00	48.4	45.0
15.1	1.63	62.8	55.0
17.8	2.16	68.0	60.0
17.7	2.38	69.9	65.1
23.2	2.92	69.5	64.7

BALANCED SINGLE ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.19,0.19,0.73 OHMS

TEST DATA

I_s	W	TORQUE	RPM	H.P.	SLIP(%)	EFF(%)	P.F(%)
13.8	630	0.0	1190	0.00	0.83	----	20.8
14.7	1380	3.8	1170	0.84	2.50	45.4	42.7
16.5	2080	7.1	1150	1.55	4.16	55.6	57.3
17.5	2420	10.3	1130	2.21	5.83	68.1	62.9
18.9	2160	11.2	1120	2.39	6.67	64.7	66.3
20.3	3090	13.9	1100	2.91	10.00	70.2	69.2

CALCULATED POINTS

I_s	H.P.	EFF(%)	P.F.(%)
12.2	0.00	----	19.6
13.7	1.00	48.4	45.0
15.1	1.63	62.8	55.0
17.8	2.16	68.0	60.0
17.7	2.38	69.9	65.1
23.2	2.92	69.5	64.7

BALANCED SINGLE ϕ - 220 V. STATOR

EXTERNAL ROTOR RESISTANCES 0.19,0.19,0.38 OHMS

TEST DATA

I_s	W	TORQUE	RPM	H.P.	SLIP(%)	EFF(%)	P.F(%)
13.8	630	0.0	1190	0.00	0.83	----	20.8
14.6	1310	3.5	1180	0.79	1.67	45.0	40.8
16.4	2060	7.4	1150	1.62	2.50	58.7	57.1
17.9	2510	10.5	1130	2.26	5.83	67.2	63.7
18.9	2800	11.3	1120	2.40	6.67	63.9	67.3
20.0	3080	13.7	1110	2.89	7.50	70.0	70.0

CALCULATED POINTS

I_s	H.P.	EFF(%)	P.F.(%)
14.6	0.00	----	16.6
15.6	1.38	46.1	38.4
15.6	1.62	67.3	55.2
19.0	2.27	71.2	66.0
19.1	2.45	70.4	65.1
20.1	3.03	68.6	74.8

BALANCED SINGLE ϕ - 220 V. STATOR

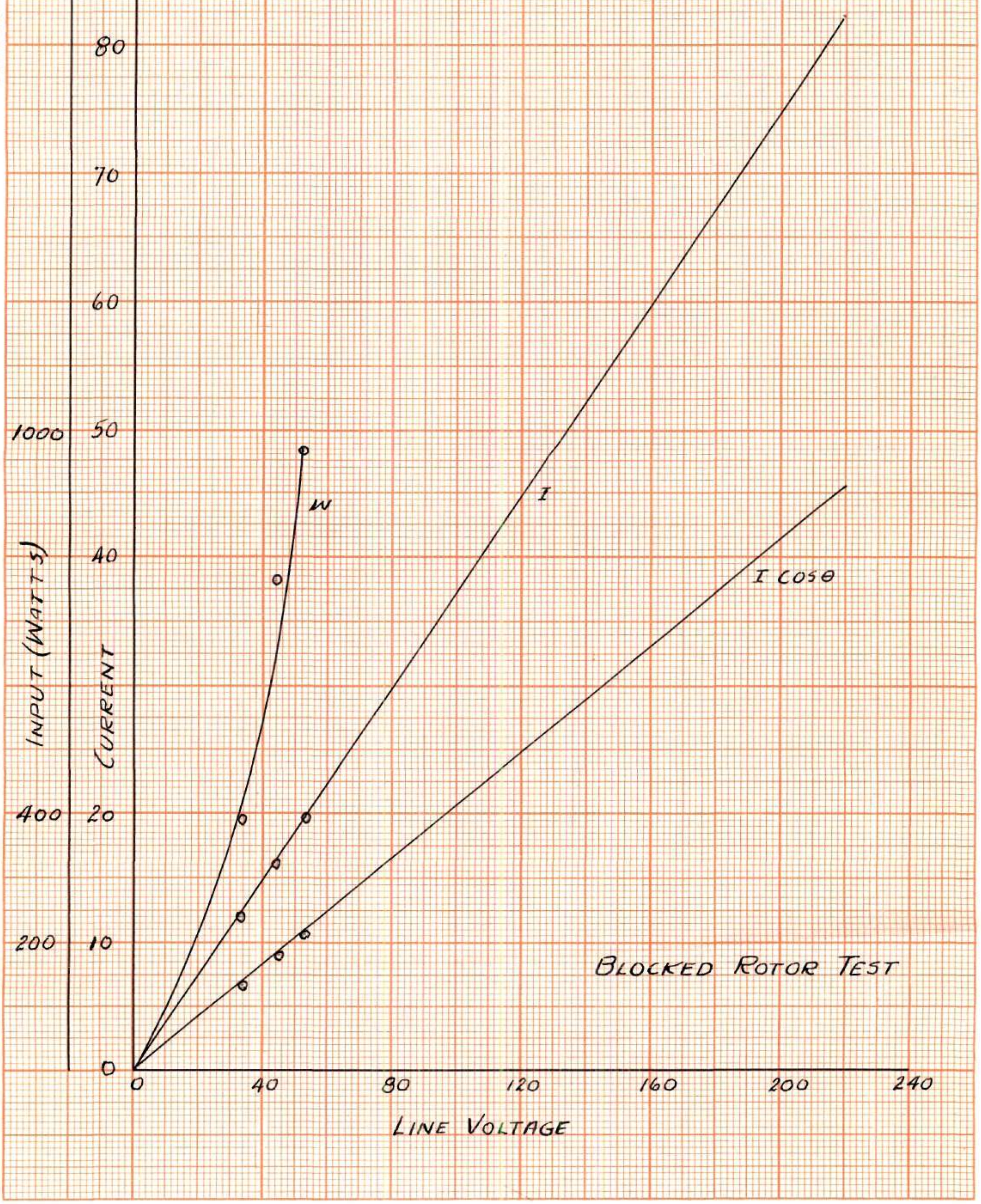
EXTERNAL ROTOR RESISTANCES 0.19,0.38,1.25 OHMS

TEST DATA

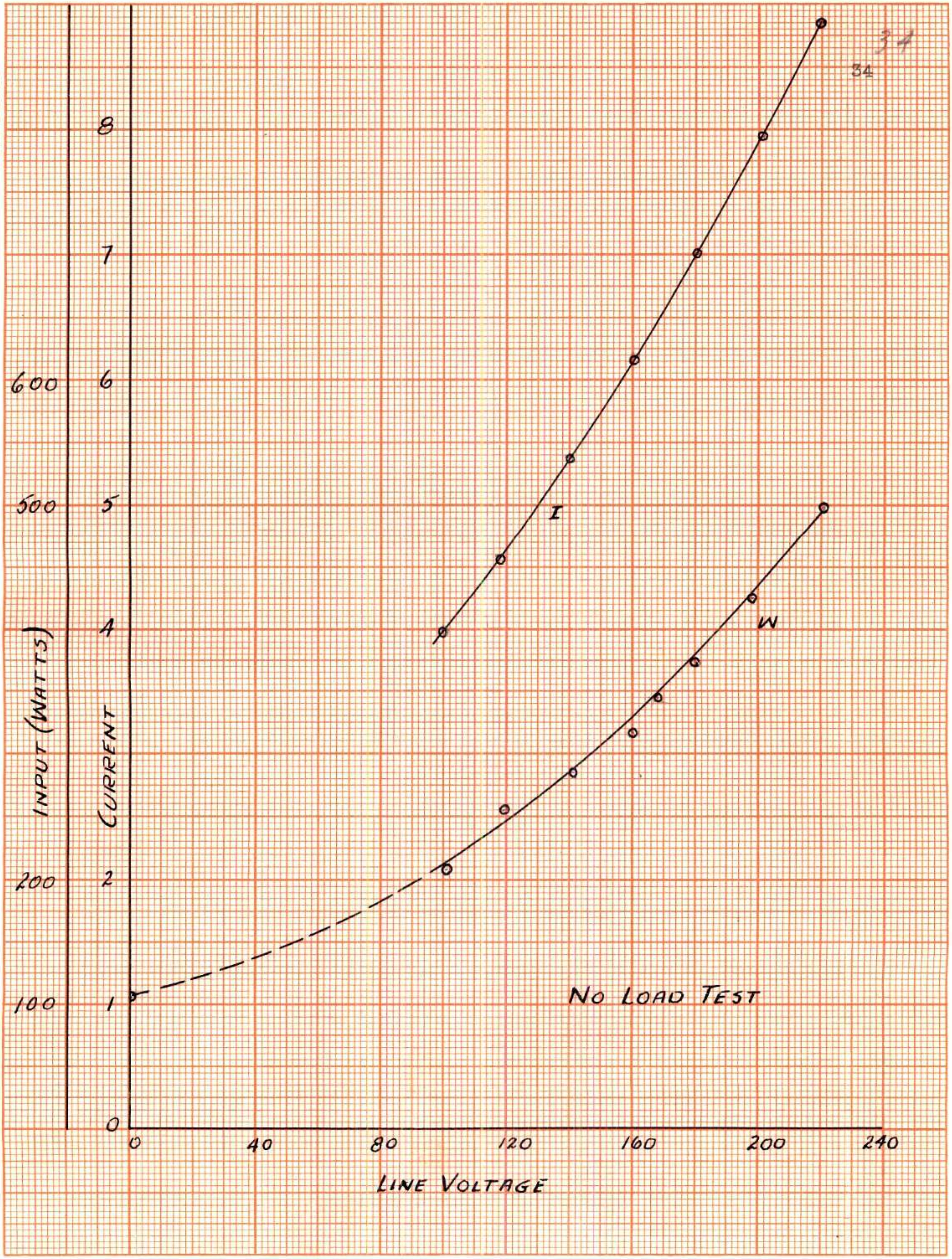
I_s	W	TORQUE	RPM	H.P.	SLIP(%)	EFF(%)	P.F(%)
13.8	630	0.0	1190	0.00	0.00	----	20.8
14.2	1050	3.1	1160	0.68	3.33	48.4	33.6
15.7	1800	6.6	1130	1.42	5.83	58.9	52.1
17.6	2380	10.0	1110	2.11	7.50	66.1	61.5
18.6	2630	11.5	1090	2.38	9.17	67.5	64.3
20.3	3070	13.7	1080	2.81	10.00	68.3	68.7

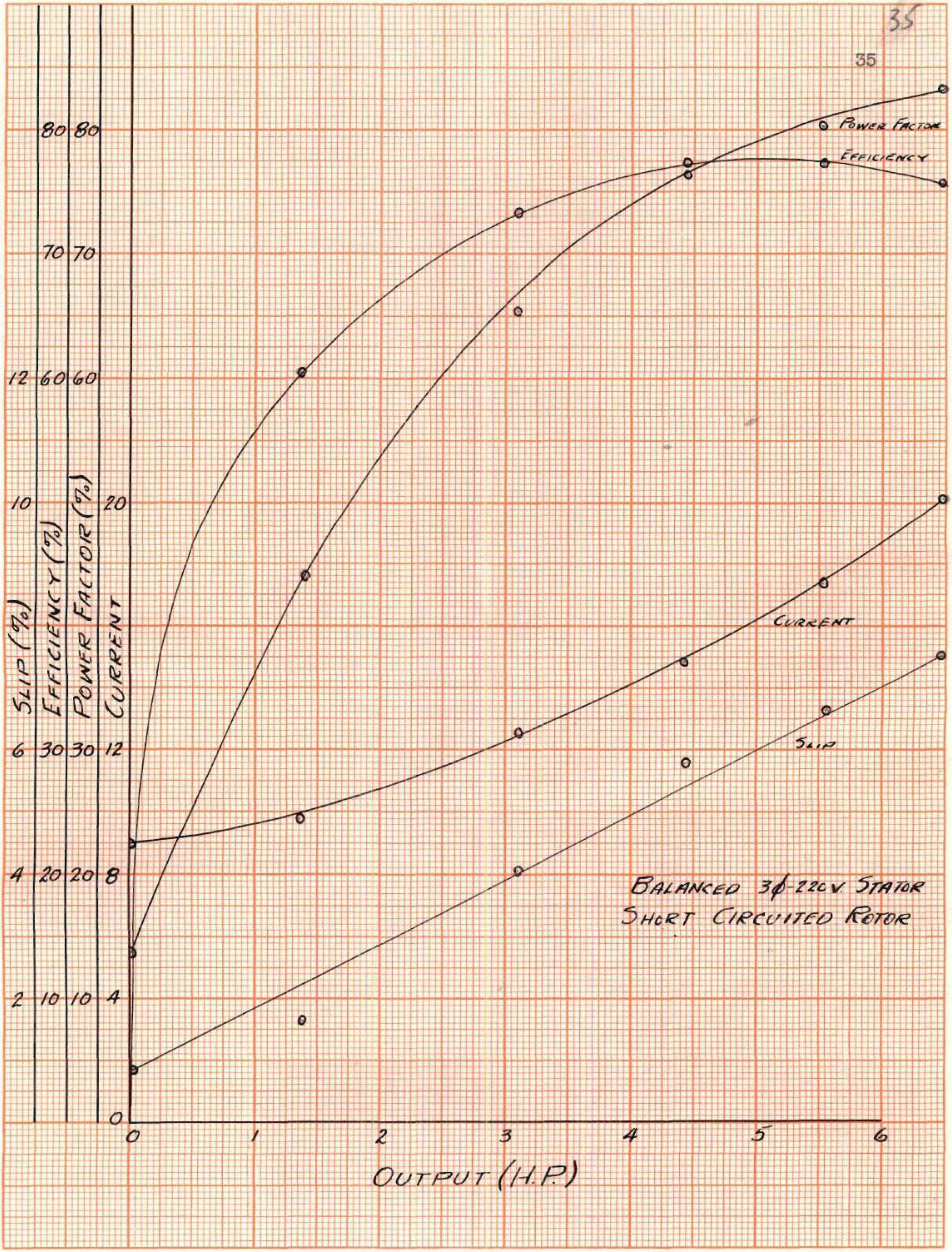
CALCULATED POINTS

I_s	H.P.	EFF(%)	P.F.(%)
12.0	0.00	----	19.8
12.2	0.75	48.7	38.6
15.0	1.48	61.4	54.5
17.9	2.15	68.6	61.7
19.3	2.47	70.3	63.0
21.4	2.95	70.2	65.2

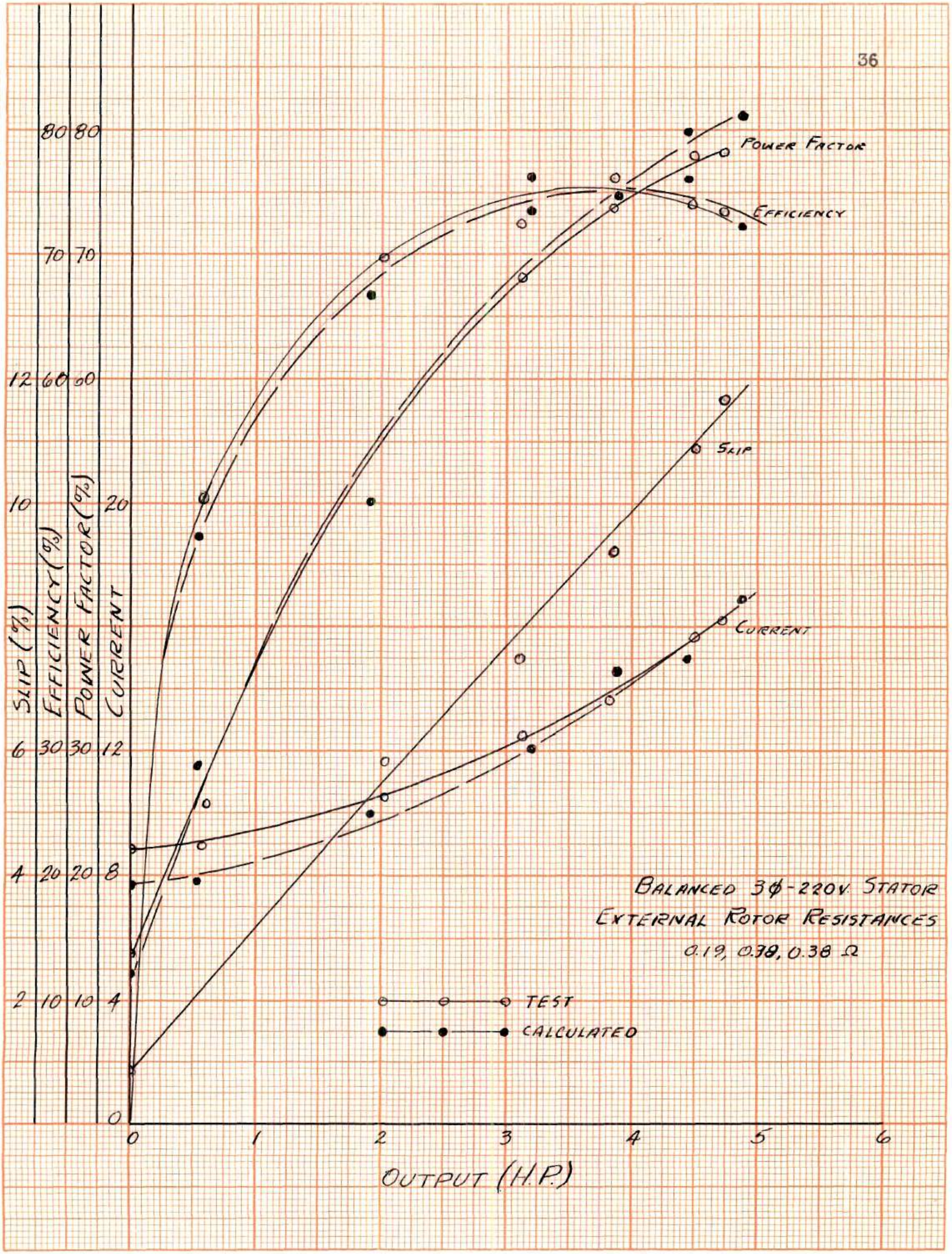


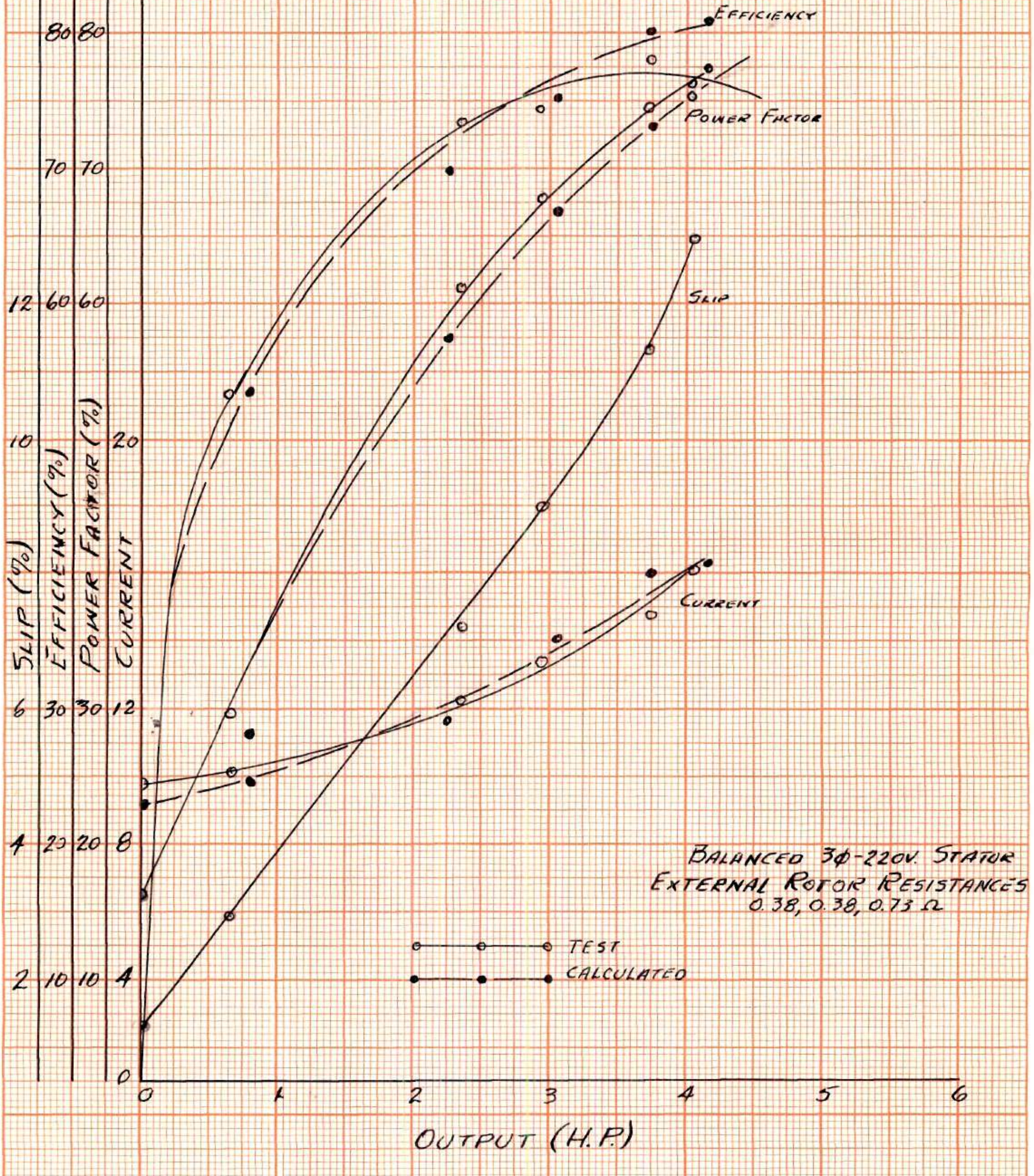
Blocked Rotor Test
Input (Watts)
Current
Line Voltage
W
I
I cos θ
80
70
60
1000
50
40
400
20
200
10
0
0
40
80
120
160
200
240


8
7
600
6
500
5
I
4
W
INPUT (WATTS)
CURRENT
200
2
100
1
NO LOAD TEST
0
0
40
80
120
160
200
240
LINE VOLTAGE

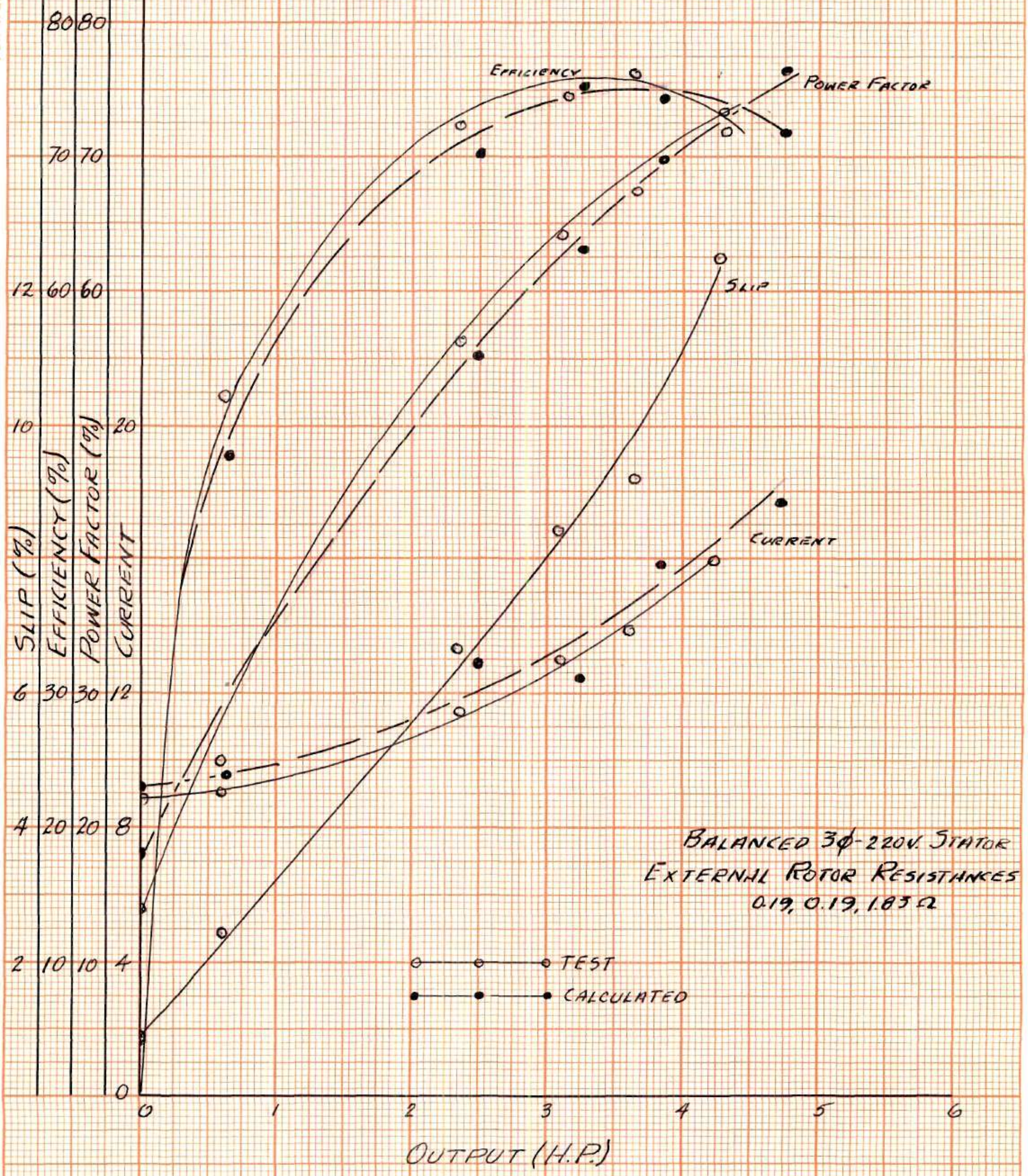


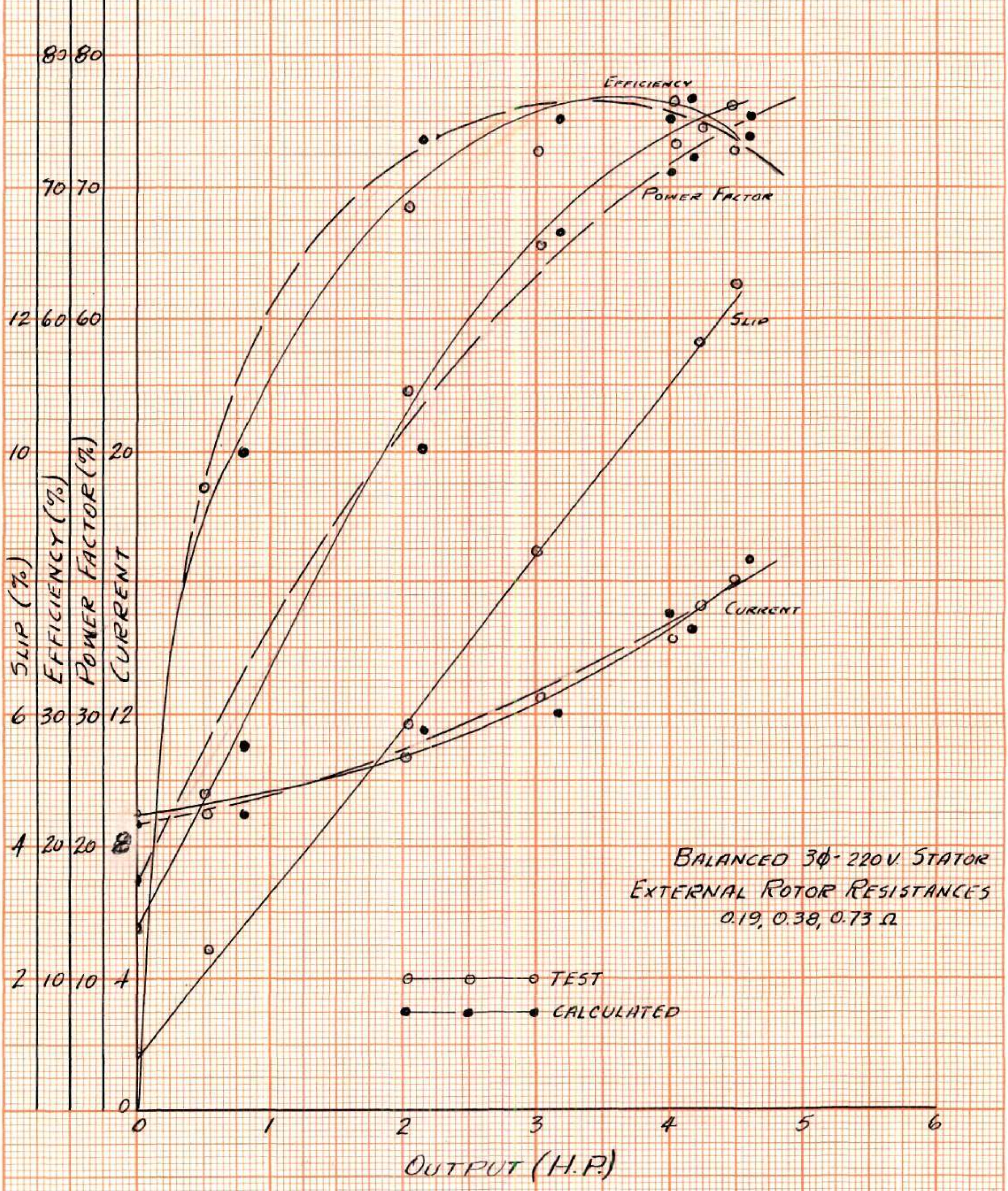
POWER FACTOR
EFFICIENCY
CURRENT
SLIP
SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
OUTPUT (H.P.)
BALANCED 3φ-220 V. STATOR
SHORT CIRCUITED ROTOR

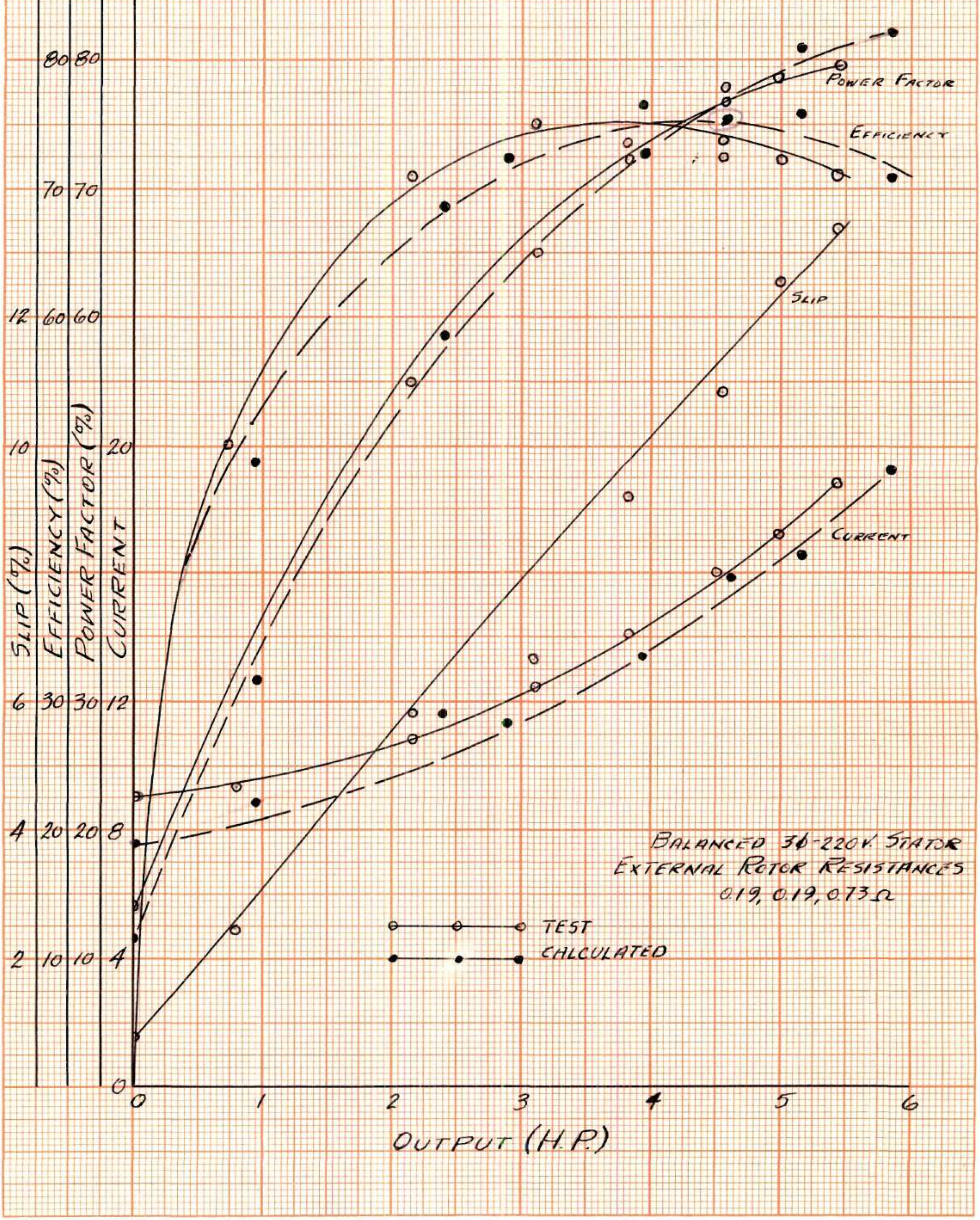

POWER FACTOR
EFFICIENCY
SLIP
CURRENT
SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
OUTPUT (H.P.)
BALANCED 3Φ-220V. STATOR
EXTERNAL ROTOR RESISTANCES
0.19, 0.38, 0.38 Ω
TEST
CALCULATED



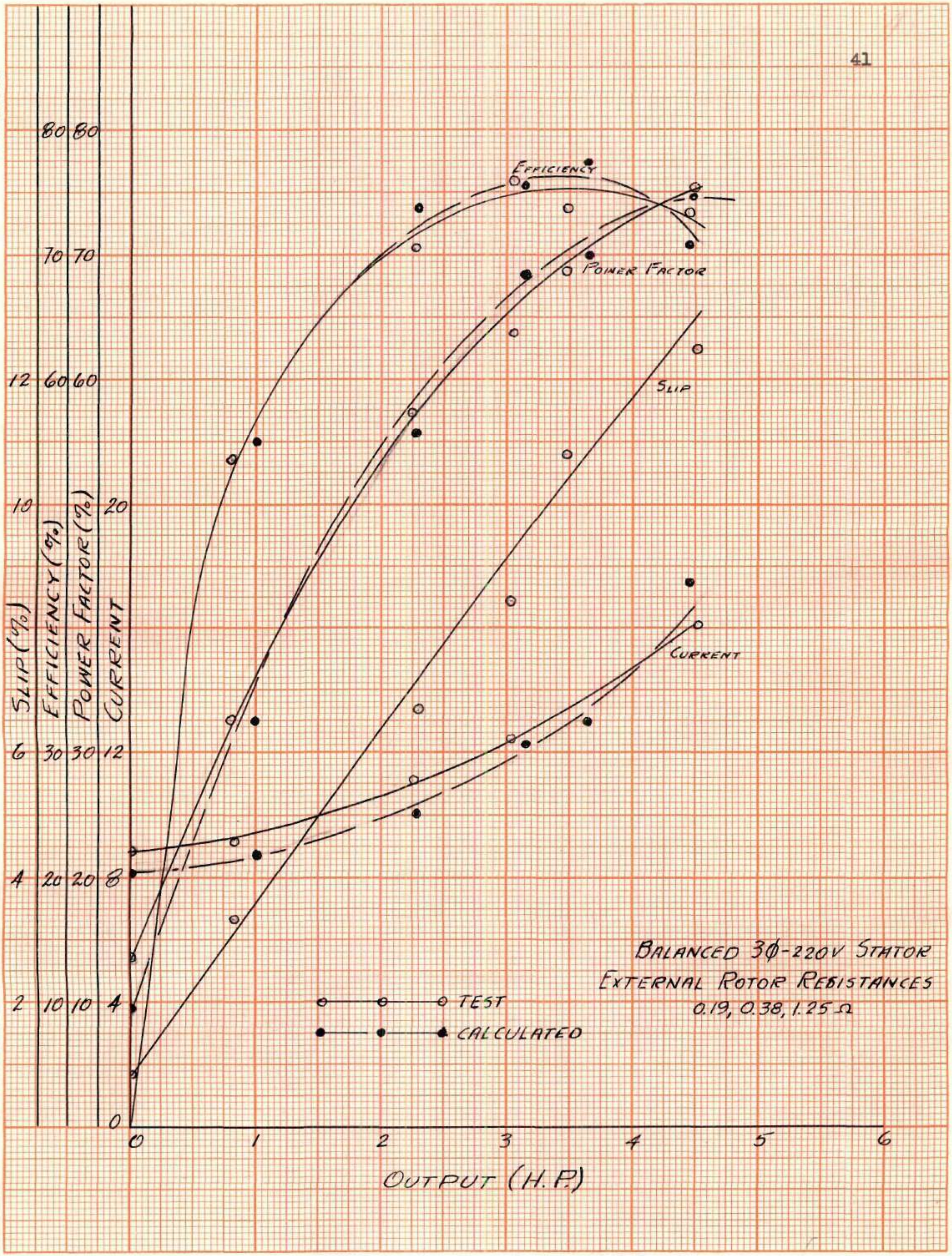
EFFICIENCY
POWER FACTOR
SLIP
CURRENT
SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
OUTPUT (H.P.)
BALANCED 3φ-220V. STATOR
EXTERNAL ROTOR RESISTANCES
0.38, 0.38, 0.73 Ω
TEST
CALCULATED

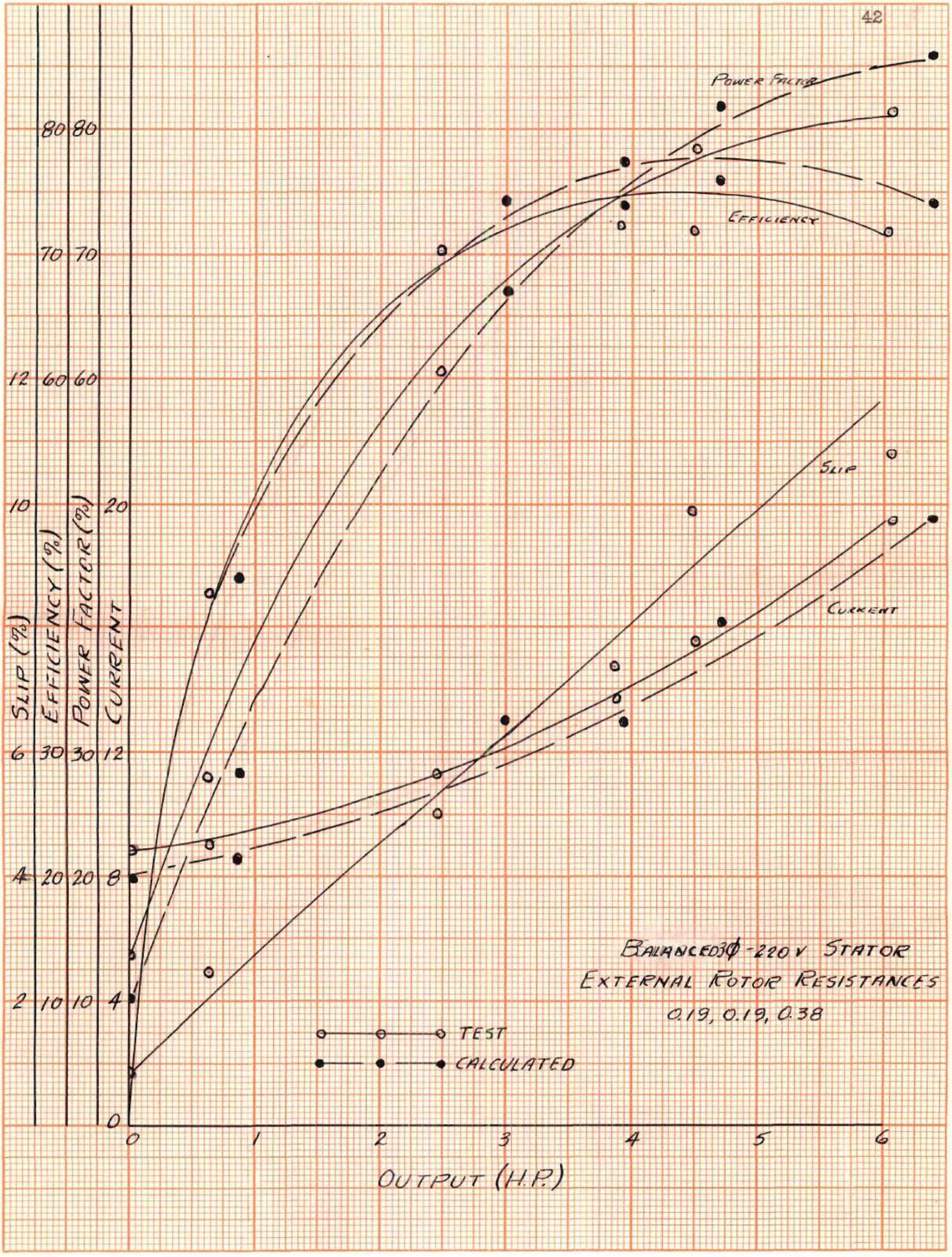

SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
EFFICIENCY
POWER FACTOR
SLIP
CURRENT
BALANCED 3φ-220V. STATOR
EXTERNAL ROTOR RESISTANCES
0.19, 0.19, 1.83 Ω
TEST
CALCULATED
OUTPUT (H.P.)

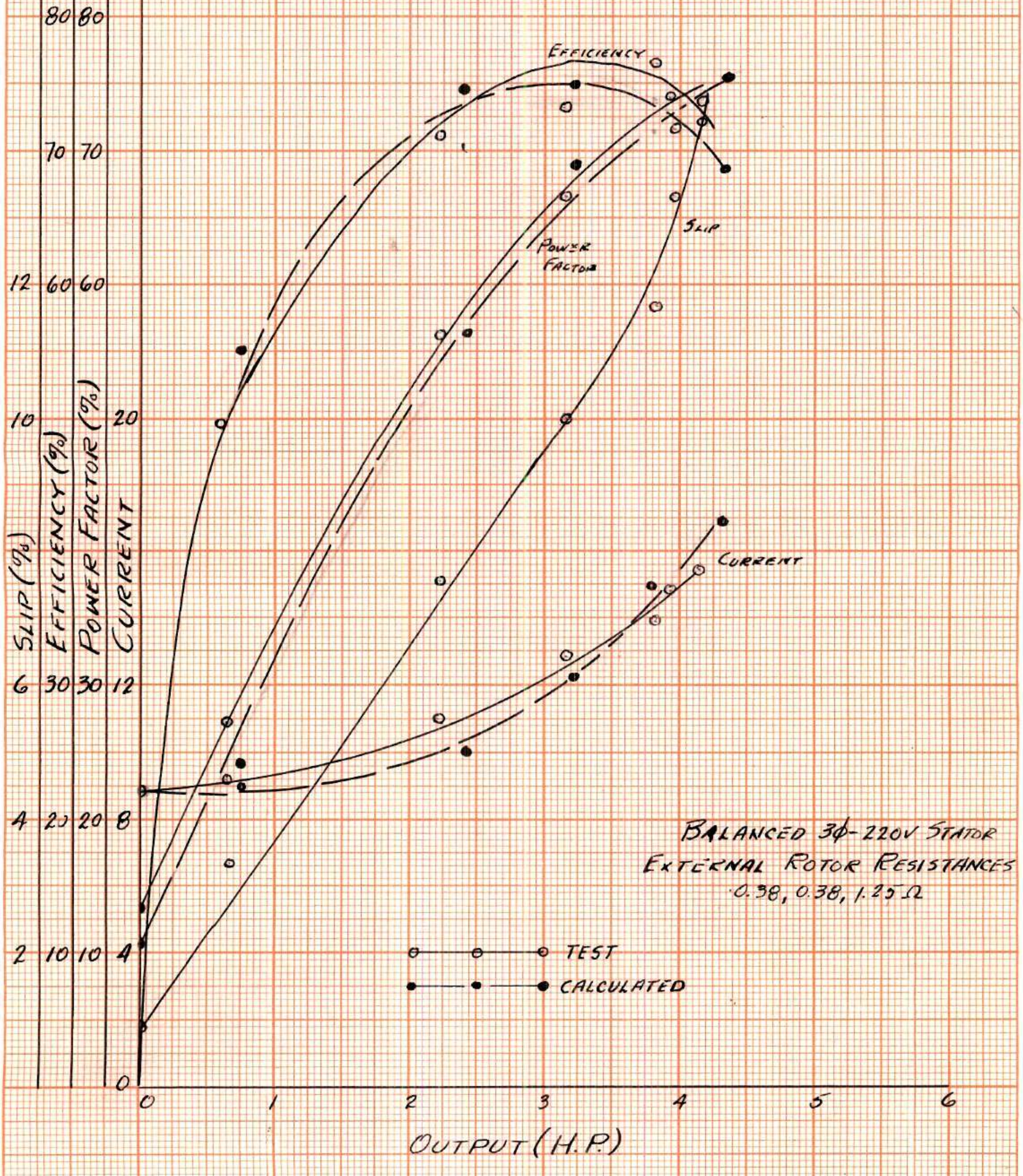

EFFICIENCY
POWER FACTOR
SLIP
CURRENT
SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
BALANCED 3φ - 220 V. STATOR
EXTERNAL ROTOR RESISTANCES
0.19, 0.38, 0.73 Ω
TEST
CALCULATED
OUTPUT (H.P.)

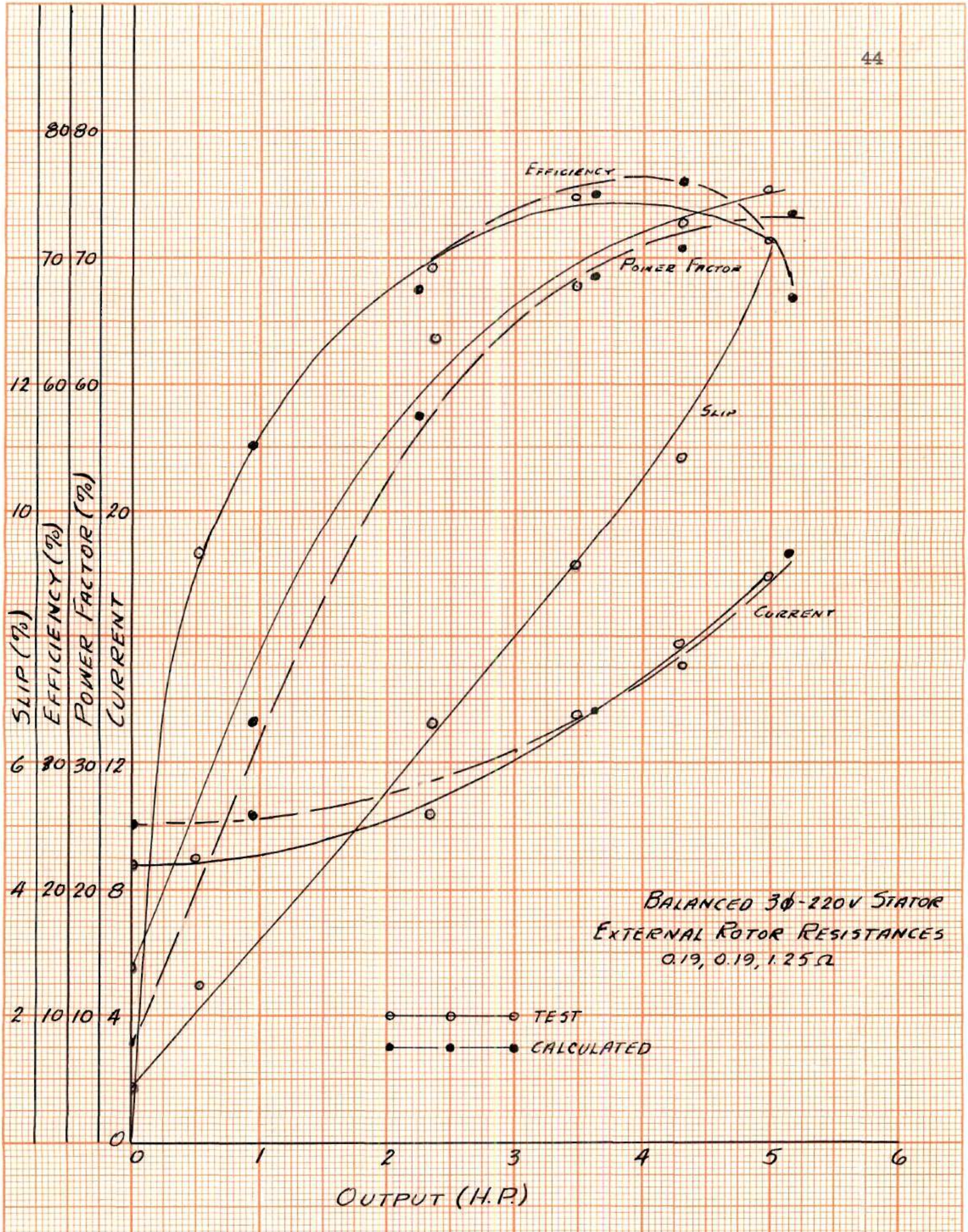


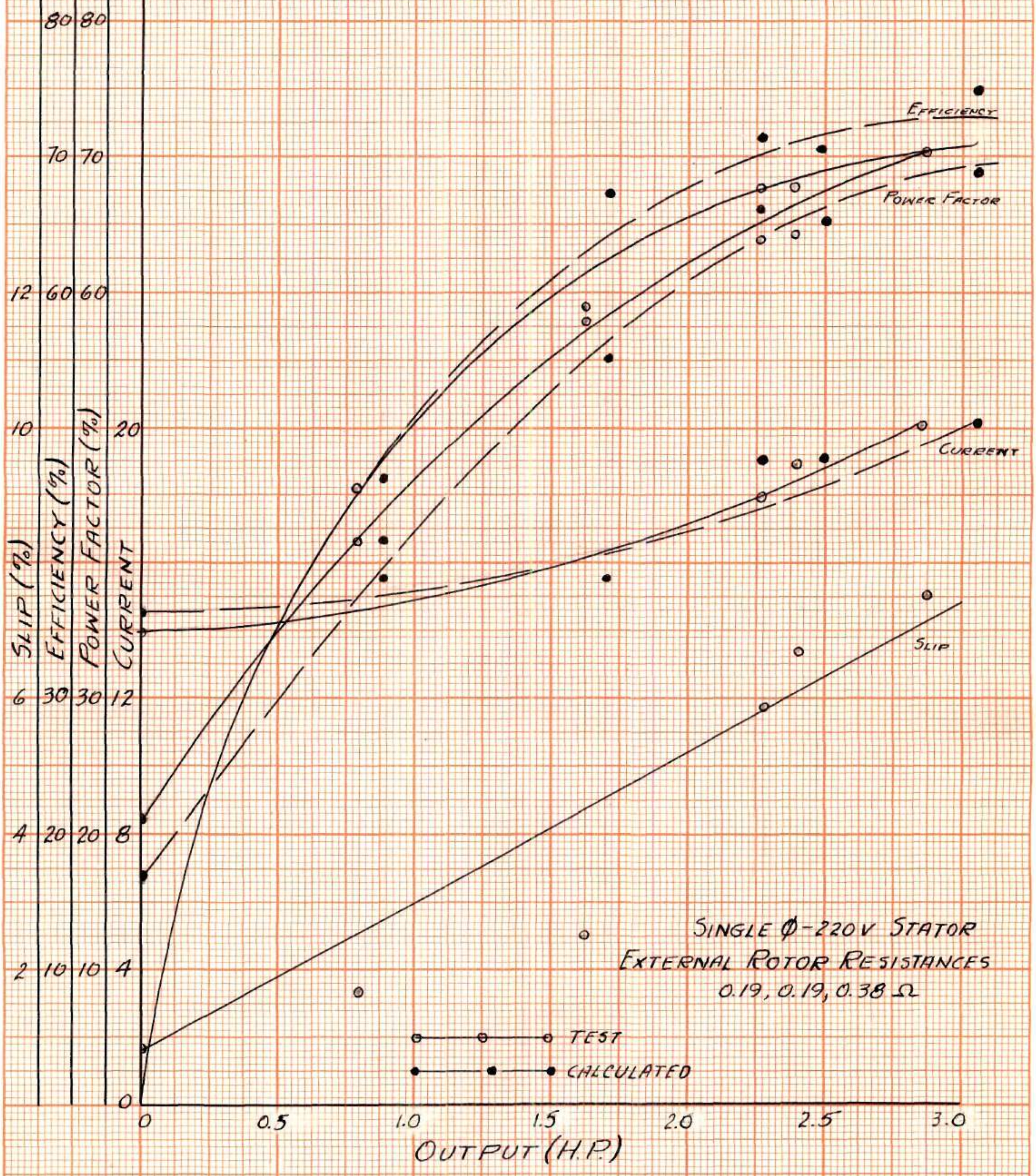
Power Factor
Efficiency
Slip
Current
Balanced 3ϕ-220V. Stator
External Rotor Resistances
0.19, 0.19, 0.73 Ω
Test
Calculated
Slip (%)
Efficiency (%)
Power Factor (%)
Current
Output (H.P.)
80 80
70 70
12 60 60
10 20
6 30 30 12
4 20 20 8
2 10 10 4
0
0
1
2
3
4
5
6


EFFICIENCY
POWER FACTOR
SLIP
CURRENT
SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
OUTPUT (H.P.)
TEST
CALCULATED
BALANCED 3φ-220V STATOR
EXTERNAL ROTOR RESISTANCES
0.19, 0.38, 1.25 Ω

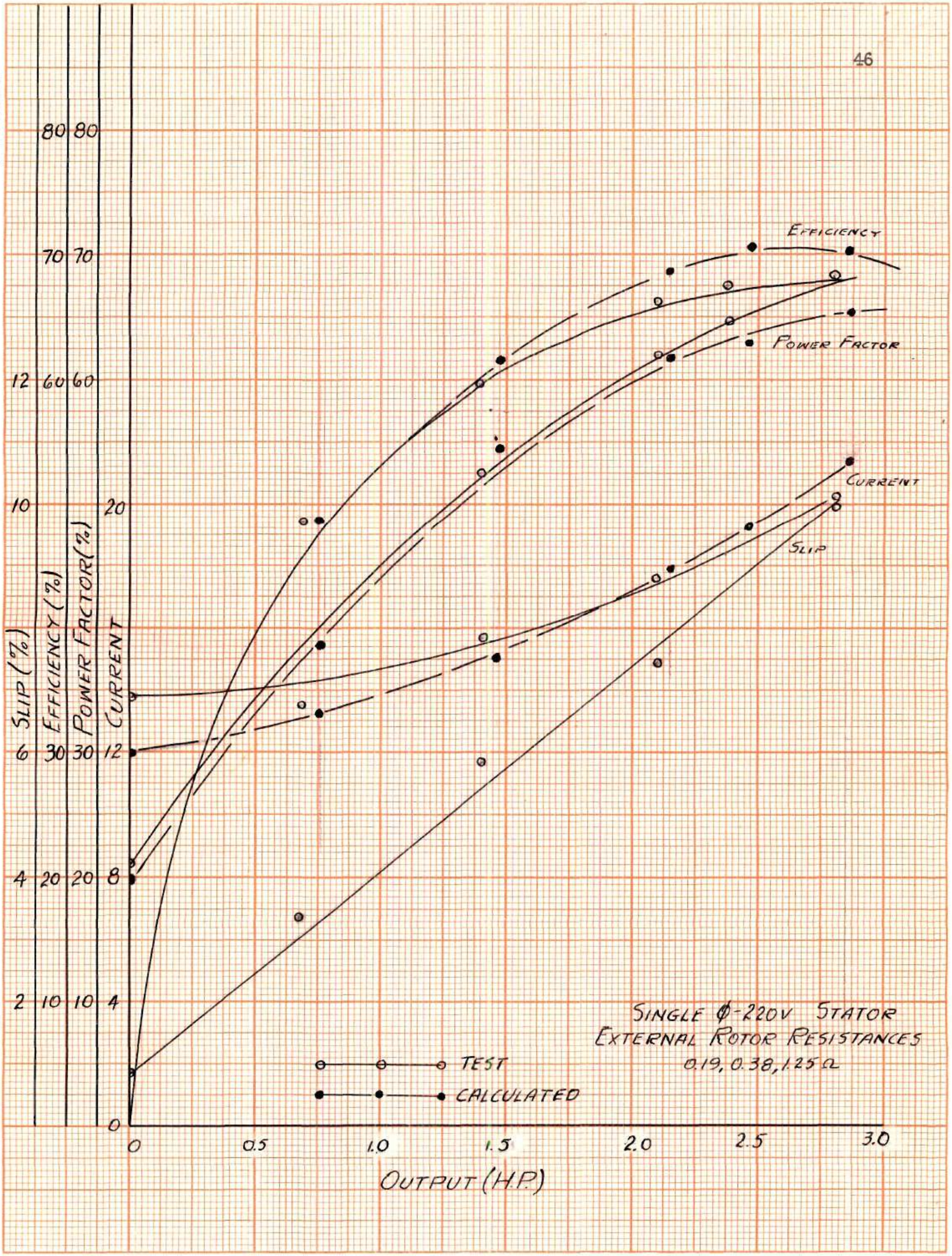

POWER FACTOR
EFFICIENCY
SLIP
CURRENT
SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
OUTPUT (H.P.)
TEST
CALCULATED
BALANCED 3Ø - 220 V STATOR
EXTERNAL ROTOR RESISTANCES
0.19, 0.19, 0.38


EFFICIENCY
POWER FACTOR
SLIP
CURRENT
SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
12 60 60
10 20
6 30 30 12
4 20 20 8
2 10 10 4
80 80
70 70
0
0 1 2 3 4 5 6
OUTPUT (H.P.)
BALANCED 3φ-220V STATOR
EXTERNAL ROTOR RESISTANCES
0.38, 0.38, 1.25 Ω
TEST
CALCULATED

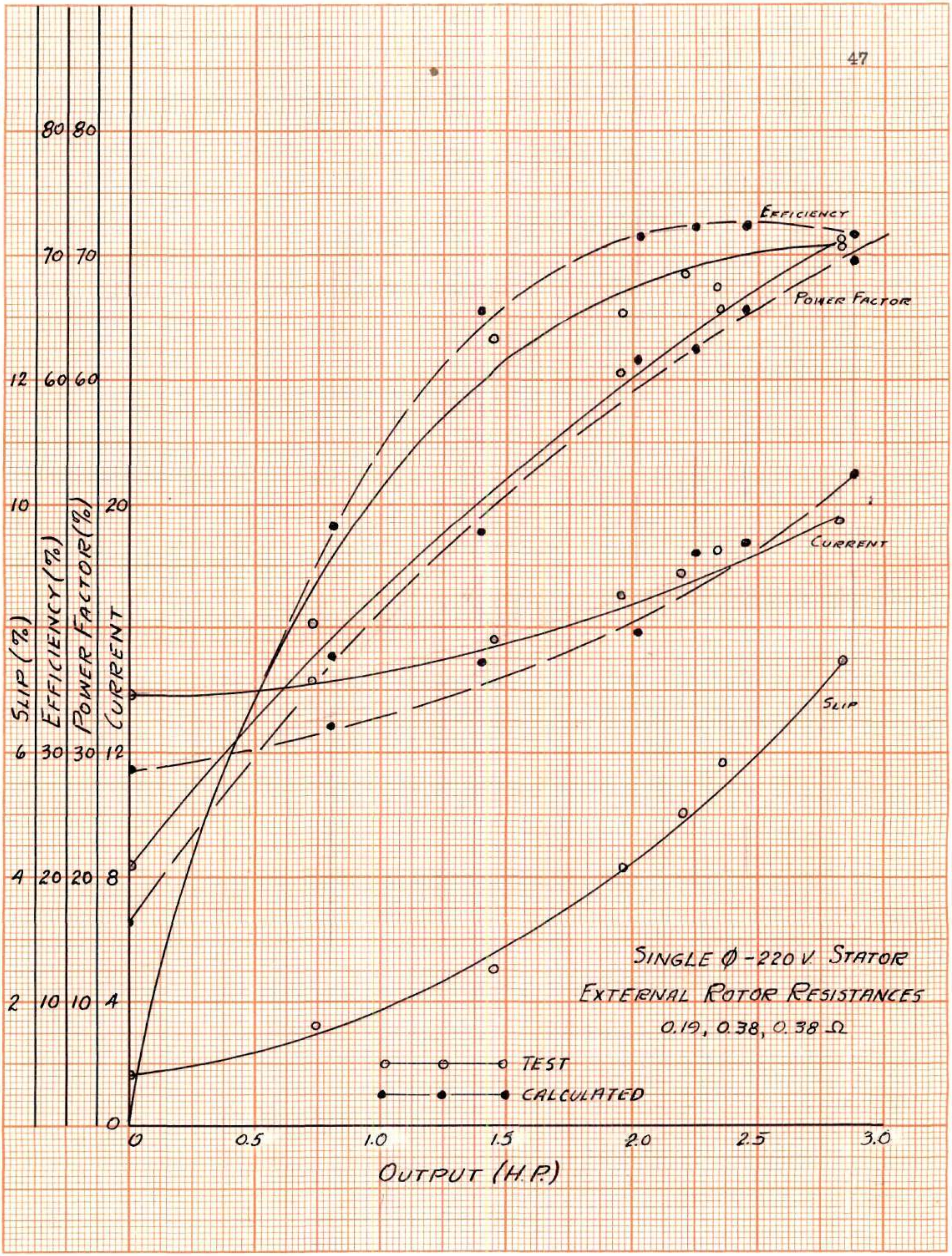

EFFICIENCY
POWER FACTOR
SLIP
CURRENT
SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
80 80
70 70
12 60 60
10 20
6 30 30 12
4 20 20 8
2 10 10 4
0
0 1 2 3 4 5 6
OUTPUT (H.P.)
BALANCED 3ϕ-220V STATOR
EXTERNAL ROTOR RESISTANCES
0.19, 0.19, 1.25 Ω
TEST
CALCULATED

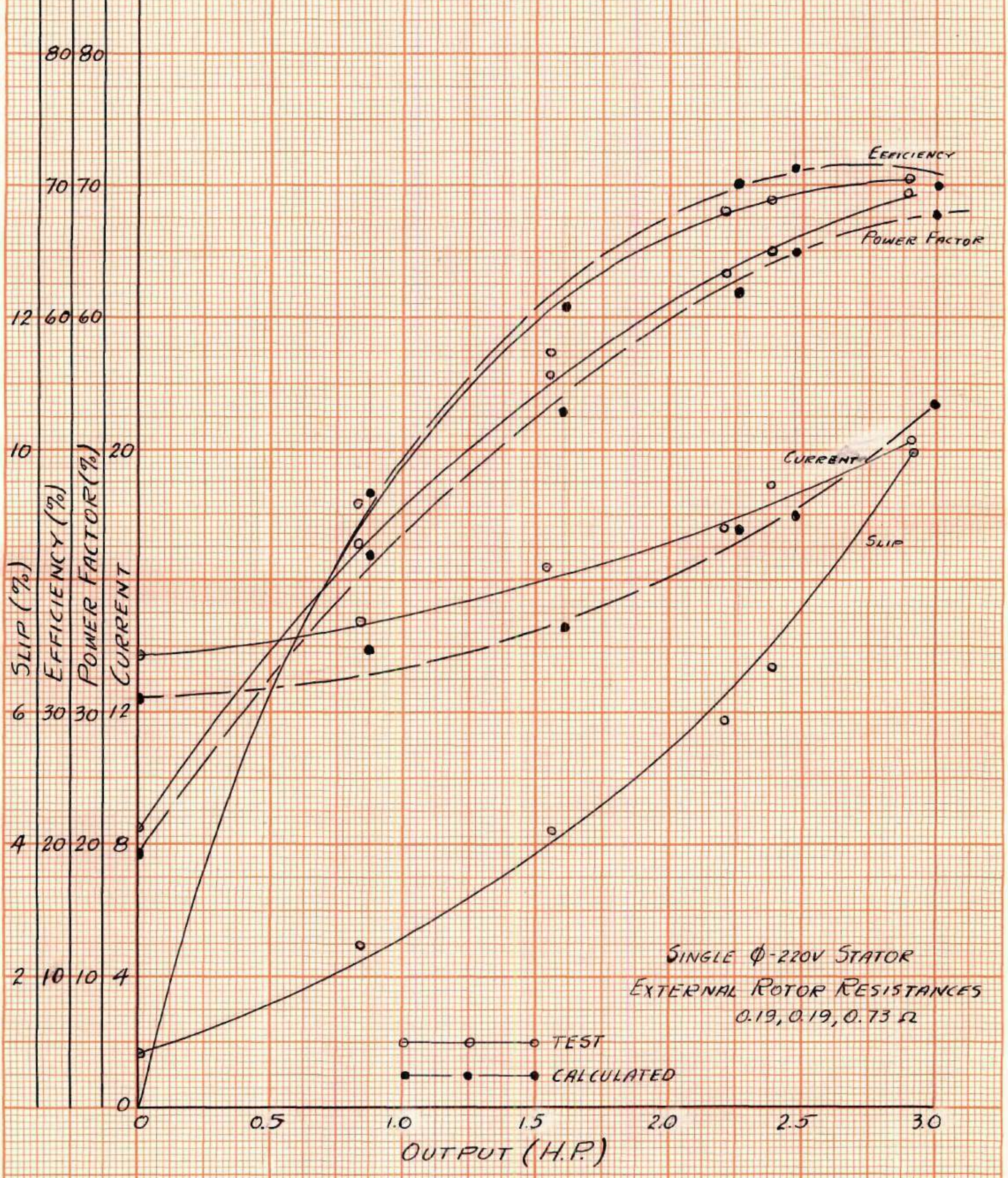


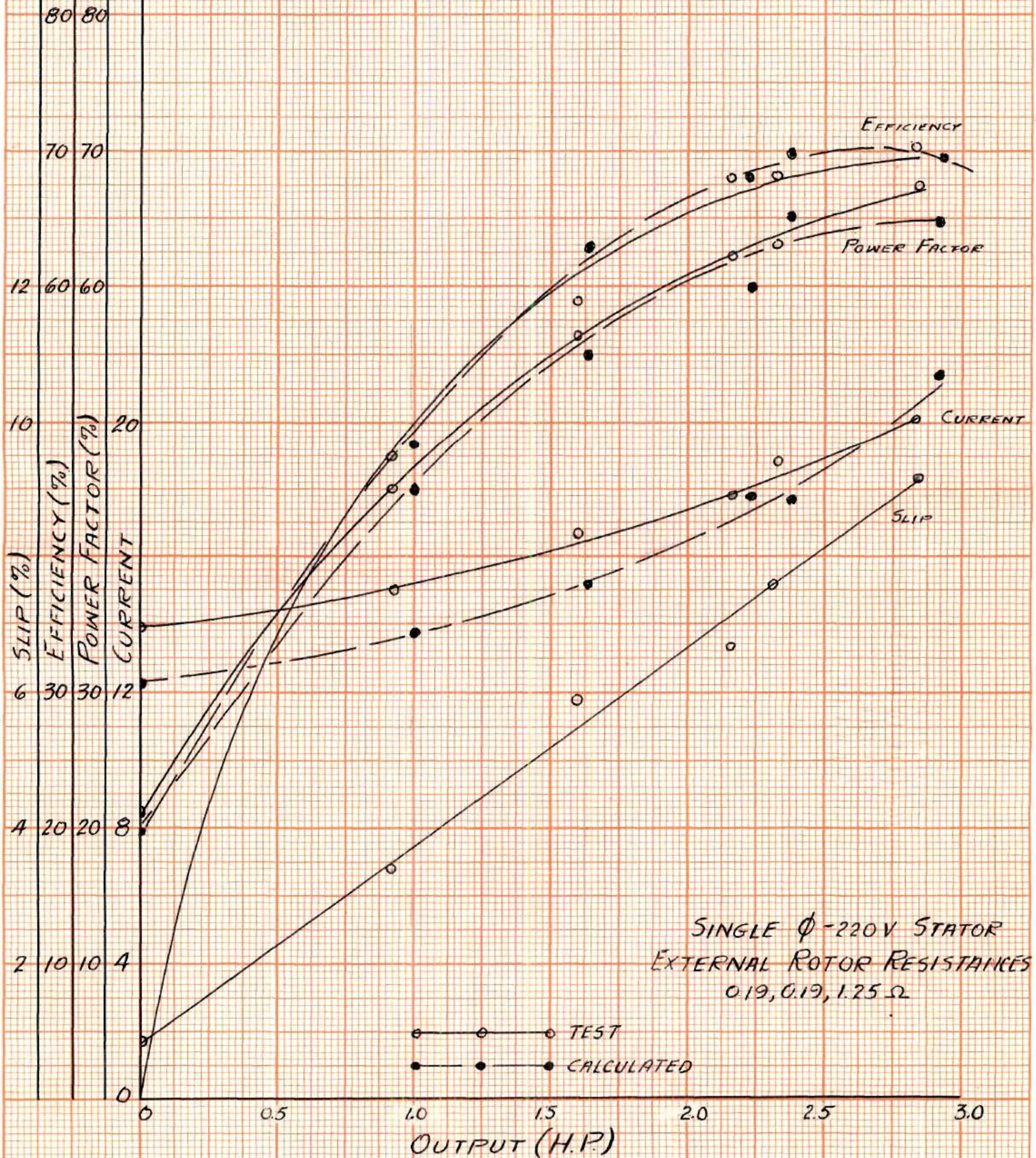
EFFICIENCY
POWER FACTOR
CURRENT
SLIP
SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
SINGLE Ø-220V STATOR
EXTERNAL ROTOR RESISTANCES
0.19, 0.19, 0.38 Ω
TEST
CALCULATED
OUTPUT (H.P.)
0
0.5
1.0
1.5
2.0
2.5
3.0

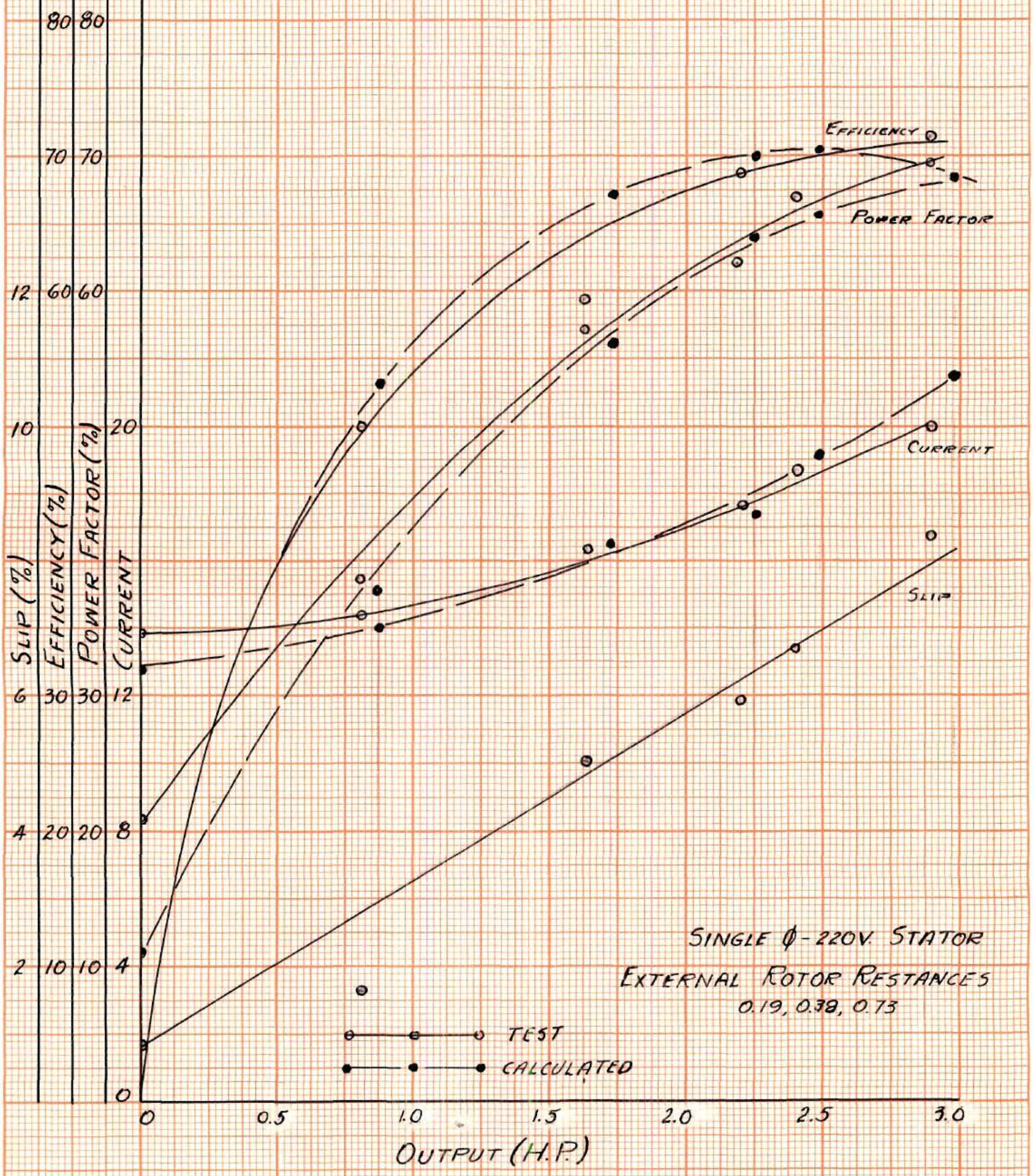


Efficiency
Power Factor
Current
Slip
Slip (%)
Efficiency (%)
Power Factor (%)
Current
Single Ø-220V Stator
External Rotor Resistances
0.19, 0.38, 1.25 Ω
Test
Calculated
Output (H.P.)
0
0.5
1.0
1.5
2.0
2.5
3.0
2
4
6
10
12
10
20
30
60
70
80
4
8
12
20


EFFICIENCY
POWER FACTOR
CURRENT
SLIP
SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
OUTPUT (H.P.)
SINGLE Ø - 220 V. STATOR
EXTERNAL ROTOR RESISTANCES
0.19, 0.38, 0.38 Ω
TEST
CALCULATED


EFFICIENCY
POWER FACTOR
CURRENT
SLIP
SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
OUTPUT (H.P.)
SINGLE Φ-220V STATOR
EXTERNAL ROTOR RESISTANCES
0.19, 0.19, 0.73 Ω
TEST
CALCULATED


EFFICIENCY
POWER FACTOR
CURRENT
SLIP
SLIP (%)
EFFICIENCY (%)
POWER FACTOR (%)
CURRENT
OUTPUT (H.P.)
SINGLE Ø - 220V STATOR
EXTERNAL ROTOR RESISTANCES
0.19, 0.19, 1.25 Ω
TEST
CALCULATED



Efficiency
Power Factor
Current
Slip
Slip (%)
Efficiency (%)
Power Factor (%)
Current
80 80
70 70
12 60 60
10 20
6 30 30 12
4 20 20 8
2 10 10 4
0
0 0.5 1.0 1.5 2.0 2.5 3.0
Output (H.P.)
Single ϕ - 220V. Stator
External Rotor Restances
0.19, 0.38, 0.73
Test
Calculated

CALCULATION OF MACHINE CONSTANTS

STATOR RES. MEASURED AT $25°C = 0.30\,\Omega$
ROTOR RES. MEASURED AT $25°C = 0.26\,\Omega$

$Zeq/\phi = \frac{Volts/\phi}{I_{sc}} = \frac{127}{83} = 1.53\,\Omega$

$Req/\phi = \frac{I_{sc}\cos\theta}{I_{sc}} \times Zeq = \frac{45.5}{83} \times 1.53 = 0.838\,\Omega$

$Xeq/\phi = \sqrt{(1.53)^2 - (0.838)^2} = 1.28\,\Omega$

EFF. RES. OF STATOR/ϕ AT $25°C$ AND $60\sim/''$

$$0.838 \times \frac{0.30}{0.26+0.30} = 0.448\,\Omega$$

OHMIC RES. OF STATOR/ϕ AT $75°C$

$$0.30(1+50\times0.00385) = 0.3578\,\Omega$$

EFF. RES. OF STATOR/ϕ AT $75°C$

$$0.3578 + 0.448 - 0.30 = 0.506\,\Omega$$

OHMIC RES. OF ROTOR/ϕ REFERED TO STATOR AT $25°C$

$$0.26 \times (1.37)^2 = 0.482\,\Omega$$

OHMIC RES. OF ROTOR REFERED TO STATOR AT $75°C$

$$0.482(1+50\times0.00385) = 0.575\,\Omega$$

EFF RES. OF ROTOR REFERED TO STATOR AT $75°C$ AND $120\sim/''$.

$$0.575 \times 1.8 = 1.035\,\Omega$$

RATIO OF TRANSFORMATION

$$a = \frac{V}{E_2}\sqrt{\frac{VE_2'}{V'E_2}} = \frac{115}{76.5}\sqrt{\frac{115}{138}} = 1.37$$

FRICTION AND WINDAGE = 105 WATTS (FROM CURVE)

CORE LOSS = $500 - 105 - 3\times0.506\times(8.9)^2 = 275$ WATTS.

SAMPLE CALCULATION FOR THREE PHASE OPERATION

EXTERNAL ROTOR RESISTANCES — $0.19\Omega, 0.19\Omega, 0.38\Omega$

$$S = \frac{1200-1070}{1200} = \frac{130}{1200} = 0.1082$$

$$I_\phi = 8.9 \overline{\backslash 82.3^\circ}$$

$$Z_{R_1} = \tfrac{1}{3}(0.19 + 0.19a + 0.38a^2) = -(0.0317 + j0.0548) = -0.0634 \angle 60^\circ$$

$$Z_{R_2} = \tfrac{1}{3}(0.19 + 0.19a^2 + 0.38a) = -0.0317 + j0.0548 = -0.0634 \overline{\backslash 60^\circ}$$

$$Z_{R_0} = \tfrac{1}{3}(0.19 + 0.19 + 0.38) = 0.253$$

$$E_{R_1} = SE_{S_1} - [SR_S + R_R + jS(x_S + x_R)]I_{R_1} - S(R_S + jx_S)I_\phi$$

$$E_{R_1} = 13.75 - [0.630 + j0.1387]I_{R_1} - [0.0548 + j0.0693]I_\phi$$

$$E_{R_1} = 13.08 + j0.401 - (0.630 + j0.1387)I_{R_1} \quad \text{——— (1)}$$

$$E_{R_2} = -\left[R_R - \frac{SR_S}{1-2S} + jS(x_S + x_R)\right]I_{R_2}$$

$$E_{R_2} = -(0.575 - 0.0698 + j0.1387)I_{R_2}$$

$$E_{R_2} = -(0.505 + j0.1387)I_{R_2} \quad \text{——— (2)}$$

$$E_{R_1} = I_{R_1}Z_{R_0} + I_{R_2}Z_{R_2}$$

$$E_{R_1} = 0.253 I_{R_1} - 0.0634 I_{R_2} \overline{\backslash 60^\circ} \quad \text{——— (3)}$$

$$E_{R_2} = I_{R_1}Z_{R_1} + I_{R_2}Z_{R_0}$$

$$E_{R_2} = -0.0634 I_{R_1} \angle 60^\circ + 0.253 I_{R_2} \quad \text{——— (4)}$$

SOLVING THESE FOUR EQUATIONS SIMULTANEOUSLY,

$$I_{R_1} = 14.8 \overline{\backslash 17.2^\circ}$$

$$I_{R_2} = 1.22 \angle 142.4$$

$$I_{S_1} = 14.8\,\overline{\angle 7.2^\circ} + 8.9\,\overline{\angle 82.3^\circ}$$

$$I_{S_1} = 19.2\,\overline{\angle 34.2^\circ}$$

$$I_{S_2} = 1.22\,\underline{\angle 42.4^\circ}$$

$$I_S = 19.2\,\overline{\angle 34.2^\circ} + 1.22\,\underline{\angle 42.4^\circ}$$

$$I_S = 19.6\,\overline{\angle 30.8^\circ}$$

$$\text{INPUT} = \sqrt{3} \times 220 \times 19.6 \times \cos 30.8^\circ$$

$$\text{INPUT} = 6410 \text{ WATTS}$$

$$\text{LOSSES} = 105 + 275 + 3(19.6)^2 \times 0.506 + 0.1082 \times 6410$$

$$\text{LOSSES} = 1657 \text{ WATTS}$$

$$\text{OUTPUT} = 6410 - 1657$$

$$\text{OUTPUT} = 4753 \text{ WATTS}$$

$$\text{DEVELOPED HORSEPOWER} = \frac{7453}{746} = 6.37$$

$$\text{EFFICIENCY} = \frac{7453}{6410} = 74.0\%$$

$$\text{POWER FACTOR} = \cos 30.8^\circ = 86.0\%$$

SAMPLE CALCULATION FOR SINGLE PHASE OPERATION

EXTERNAL ROTOR RESISTANCES — $0.19\Omega, 0.19\Omega, 0.38\Omega$.

$$I_\phi = 13.8\angle 78°0 \qquad S = \frac{1200-1110}{1200} = 0.075$$

$$I_{\phi_1} = 8.9\angle 82°3$$

$$I_{\phi_2} = 13.8\angle 78°0 - 8.9\angle 82°3$$

$$I_{\phi_2} = 4.99\angle 70°4$$

$$E_{S_1} = \tfrac{1}{3}(110 + a^2 \times 110\angle 180°) = 55 + j\,31.4$$

$$E_{S_2} = \tfrac{1}{3}(110 + a \times 110\angle 180°) = 55 - j\,31.4$$

$$Z_{R_1} = \tfrac{1}{3}(0.19 + 0.19a + 0.38a^2) = -(0.0317 + j0.0548) = -0.0634\angle 60°$$

$$Z_{R_2} = \tfrac{1}{3}(0.19 + 0.19a^2 + 0.38a) = -0.0317 + j0.0548 = -0.0634\angle 60°$$

$$Z_{R_0} = \tfrac{1}{3}(0.19 + 0.19 + 0.38) = 0.253$$

$$E_{R_1} = SE_{S_1} - [SR_S + R_R + jS(x_S + x_R)]I_{R_1} - S(R_S + jx_S)I_{\phi_1}$$

$$E_{R_1} = 4.12 + j2.36 - [0.613 + j0.096]I_{R_1} - 0.0612\angle 51°7 \times 8.9\angle 82°3$$

$$E_{R_1} = 3.65 + j2.64 - (0.613 + j0.096]I_{R_1} \quad \text{— (1)}$$

$$E_{R_2} = SE_{S_2} - \left[\left(R_R - \frac{SR_S}{1-2S}\right) + jS(x_S + x_R)\right]I_{R_2} + S\left[\frac{R_S}{1-2S} - jx_S\right]I_{\phi_2}$$

$$E_{R_2} = 4.12 - j2.36 - [(0.575 - 0.045) + j0.096]I_{R_2} + 0.075[0.595 - j0.64]I_{\phi_2}$$

$$E_{R_2} = 3.97 - j2.07 - (0.530 + j0.096)I_{R_2} \quad \text{— (2)}$$

$$E_{R_1} = Z_{R_0}I_{R_1} + Z_{R_2}I_{R_2}$$

$$E_{R_1} = 0.253I_{R_1} - 0.0634\,I_{R_2}\angle 60° \quad \text{— (3)}$$

$$E_{R_2} = Z_{R_1}I_{R_1} + Z_{R_0}I_{R_2}$$

$$E_{R_2} = -0.0634I_{R_1}\angle 60° + 0.253\,I_{R_2} \quad \text{— (4)}$$

SOLVING THESE FOUR EQUATIONS SIMULTANEOUSLY,

$$I_{R_1} = 6.95 + j\,3.02$$

$$I_{R_2} = 5.25 - j\,2.92$$

$$I_R = I_{R_1} + I_{R_2} = 12.20 + j\,0.09$$

$$I_S = I_R + I_\phi = 15.07 - j\,13.41$$

$$I_S = 20.1 \,\overline{|41.^\circ 6}$$

$$\text{INPUT} = 220 \times 20.1 \times \cos 41.^\circ 6$$

$$\text{INPUT} = 3300 \text{ WATTS}$$

$$\text{LOSSES} = 105 + 275 + 2(20.1)^2 \times 0.506 + 0.075 \times 3300$$

$$\text{LOSSES} = 1036 \text{ WATTS}$$

$$\text{OUTPUT} = 3300 - 1036 = 2264$$

$$\text{DEVELOPED HORSEPOWER} = \frac{2264}{746} = 3.03$$

$$\text{EFFICIENCY} = \frac{2264}{3300} = 68.6\%$$

$$\text{POWER FACTOR} = \cos 41.^\circ 6 = 74.8\%$$